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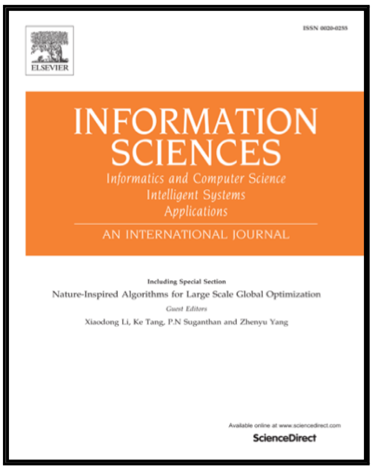

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Observer-Based Adaptive Neural Network Controller for Uncertain Nonlinear Systems with Unknown Control Directions Subject to Input Time Delay and Saturation

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Abstract

This paper addresses the design of an observer based adaptive neural controller for a class of strict-feedback nonlinear uncertain systems subject to input delay, saturation and unknown direction. The input delay has been handled using of an integral compensator term in the controller design. A neural network observer has been developed to estimate the unmeasured states. In the observer design, the Lipschitz condition has been relaxed. To solve the problem of unknown control directions, the Nussbaum gain function has been applied in the backstepping controller design. "The explosion of complexity" occurred in the traditional backstepping technique has been avoided utilizing the dynamic surfaces control (DSC) technique and the designed controller is singularity free. It has been shown that all closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB) and the output tracking error converges to a small neighborhood of the origin by choosing the design parameters appropriately. The numerical examples illustrate the effectiveness of the proposed control scheme.

Keywords: Adaptive neural network; Dynamic surface control; Input delay; Input saturation; Neural network observer; Unknown control direction.

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1. Introduction

During the past decades, control design of nonlinear systems has attracted numerous researchers' interests. Adaptive control based on the backstepping technique is one of the most common strategies for a large class of nonlinear systems [40]. Nevertheless, the most common drawback of backstepping technique is "the explosion of complexity", resulting from differentiating the virtual controllers repetitively. Particularly, as order of the system increases, the complexity grows extremely [29]. This problem can be solved using the DSC technique [19, 20, 22, 36, 38, 49]; however, in the case of model mismatch, the controller design can be much more difficult. To deal with this problem, neural network (NN) and fuzzy logic systems (FLS) are used to approximate the unknown system dynamics [6]. In [8, 18, 19, 30, 31, 33, 37, 39, 46], adaptive intelligent output feedback controllers have been proposed for the nonlinear strict-feedback systems. In [2], the sliding-mode control of nonlinear uncertain systems with unmolded actuator dynamics has been considered.

Input time delay is frequently encountered in practical systems such as hydraulic systems and chemical processes. Input delay can lead to instability of the closed-loop system and degradation of the controller performance. In [45, 50], an adaptive fuzzy feedback controller design for nonlinear single-input-single-output (SISO) systems with input delay has been addressed. In [17], an adaptive fuzzy tracking controller for nonlinear systems with input delay has been designed utilizing the Pade approximation. Zhu et al. [51], extended this approach to control the multi-input-multi-output (MIMO) nonlinear systems with input delay.

In most practical control applications, usually some of the states are unavailable for measurement and the system dynamics are not completely known. In such cases, the NN or FLS observers can be used to estimate the unmeasured states of the systems with unknown dynamics [32]. Adaptive fuzzy observer for control of SISO nonlinear systems has been proposed in [16, 35, 21]. In [7], design of an adaptive NN controller for the non-strict-feedback nonlinear systems in the absence of full state measurements has been addressed. In most observer designs, the Lipschitz condition for the unknown nonlinear functions describing the system dynamics should be satisfied. The stability analysis can be independent of the Lipschitz condition if fuzzy logic systems or neural networks are used in the design of adaptive observers [42].

Control of many industrial systems is faced with input constraints such as dead zone, backlash, or input saturation. Among these system input nonlinearities, input saturation is one of the most important constraints which severely restricts the system performance and can lead to instability [21]. In [48], an augmented system which has the same order as the nonlinear system, is incorporated in the controller design to compensate the effect of input saturation. An innovative approach for design of a robust adaptive controller for uncertain nonlinear systems in the presence of input saturation and external disturbances has been proposed in [25, 41].

In designing the adaptive neural network controllers, control singularity problem might occur. If the system dynamics are not known, these dynamics should be estimated. When the estimated values of unknown dynamics approach zero, controller singularity can occur [27]. To deal with this problem, some approaches have been proposed in the literature namely; the projection algorithm [10], the integral Lyapunov function method [47], the coordinate transformation method [15], and the direct adaptive controller technique [12]. In this work a novel approach has been proposed to avoid the controller singularity problem.

Additionally, in some control applications, the control directions are not known. When there is no a priori knowledge of the virtual control coefficients, controller design becomes much more difficult. This problem has been solved using different approaches such as utilizing the Nussbaum function [29, 34], estimating the unknown parameters including the unknown control direction [14], and using the correction-vector method [5]. The Nussbaum gain function has been used in the design of adaptive controllers for nonlinear systems with unknown control directions in [11, 13, 29].

Some of the aforementioned restrictions are considered by other researchers. For example, in [21, 23] an adaptive fuzzy output feedback controller for nonlinear systems in the presence of input saturation has been proposed. In another work [9], an observer-based adaptive NN controller is proposed for a class of uncertain nonlinear systems in the presence of input saturation. In these works, it has been assumed that the control directions are known a priori which is not true in many control applications. In [29], an adaptive neural network controller is designed for a class of uncertain nonlinear strict-feedback systems with unknown control directions in the presence of input saturation. In the mentioned work, the authors assumed that all states are available for measurement which is not a realistic assumption from a practical point of view. In [1], an observer-based adaptive fuzzy output feedback controller for a class of nonlinear system with unknown control directions and input saturation has been proposed, but the input delay and "explosion of complexity", occurring in the conventional backstepping controller design, are not taken into account.

To the best of authors' knowledge, there is no published work that considers simultaneously all of the aforementioned restrictions, i.e. unmeasured states, input saturation, input delay, controller singularity problem and unknown control directions for a class of uncertain SISO nonlinear systems. In this work, a controller for such systems has been proposed. In addition "the explosion of complexity" which occurs in the traditional backstepping control strategy has been avoided using the dynamic surfaces control technique.

The main contributions of the present work are summarized below.

1- In the development of the proposed control scheme, the Lipschitz condition which is a restrictive assumption in the design of the NN adaptive observer has been relaxed, while this assumption has been made in several related works [21, 50].

2- The proposed method for handling the singularity problem, avoids the complexities in stability analysis presented in [47]. Additionally, compared to the approaches proposed in [11, 13, 29], the presented method is simpler and depends only on one Nussbaum Nussbaum function which leads to less oscillation in the control action.

3- Through the DSC technique, "the explosion of complexity" occurring in the conventional backstepping controller design has been avoided. This advantage becomes more highlighted when the uncertain nonlinear system has high order dynamics.

4- The proposed method is able to tackle the effect of time-varying input delay, while in [45, 50], it has been assumed that the input delay is constant.

5- Using the Nussbaum gain function is one of the most common approaches for handling the unknown control directions, but the control action becomes oscillatory as the number of Nussbaum functions increases. In the present study, by utilizing a change of coordinate, only one Nussbaum function has been used which leads to fewer oscillations in the control action.

6- Stability of the closed-loop system and boundedness of all signals have been established in the presence of input saturation, observer dynamics and input delay.

The paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. In Section 3, design of the Luenberger-like observer is addressed. Design of the NN adaptive controller based on the backstepping technique and stability analysis are presented in Section 4. Effectiveness of the proposed scheme has been demonstrated via simulation study in Section 5. Conclusion is drawn in Section 6. Finally, the future works are discussed in Section 7.

2. Problem formulation and preliminaries

2.1 Control problem

Consider the following uncertain nonlinear strict feedback system with time-varying input delay, unknown control directions subject to external disturbances as given below

$$\begin{cases} \dot{x}_i = g_i x_{i+1} + f_i(\underline{x}_i) + d_i(t) & 1 \le i \le n-1 \\ \dot{x}_n = G_n(\underline{x}_n)u(t-\tau) + f_n(\underline{x}_n) + d_n(t) \\ y = x_1 \end{cases} \tag{1}$$

where $\underline{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$ are the state variables and it is assumed that for $i \ge 2$, x_i are unavailable. $g_i, 1 \le i \le n-1$ are the unknown control coefficients. $f_i(.), 1 \le i \le n$ and $G_n(.)$ are the unknown nonlinear smooth functions. $d_i(.), 1 \le i \le n$ are the unknown bounded disturbances and $\tau \ge 0$ is the input time delay. $y \in R$ and $u \in R$ are output and input of the system, respectively. Due to input saturation, the system input, u, is related to the designed control signal $v \in R$ as given below

$$u(t) = Sat(v) = \begin{cases} u_m Sign(v(t-\tau)), & |v(t-\tau)| \ge u_m \\ v(t-\tau) & , \ |v(t-\tau)| < u_m \end{cases} \tag{2}$$

where $Sat(.)$ and $Sign(.)$ denote saturation and unit sign function, respectively. u_m is the known magnitude of the saturation limit. The control objective is designing an adaptive controller for the above nonlinear system in the presence of the aforementioned restrictions.

Remark 1. $G_n(.)$ is a nonzero function with unknown sign which can be expressed as follows

$$G_n(\underline{x}_n) = H(\underline{x}_n) g_n \tag{3}$$

where $H(\underline{x}_n) = abs\left(G_n(\underline{x}_n)\right)$ and $g_n = sign\left(G_n(\underline{x}_n)\right)$. Utilizing the above equation and the following transformation are two important steps in simplifying the controller design and minimizing the number of the Nussbaum functions used for the design of the controller. Existence of the unknown control coefficients, $g_i, 1 \le i \le n$, in the system (1), makes the controller design much more difficult. In order to simplify the controller design, the approach proposed in [44] has been used. Define the new state variables $X_i = \frac{x_i}{\prod_{j=i}^{n} g_j}, 2 \le i \le n$, the new smooth functions $F_i(\underline{X}_i) = \frac{f_i(\underline{x}_i)}{\prod_{j=i}^{n} g_j}, 2 \le i \le n$ and the new disturbances $D_i(t) = \frac{d_i(t)}{\prod_{j=i}^{n} g_j}, 2 \le i \le n$. The first state variable, X_1, the first smooth function, $f_1(x_1)$, and the first disturbance, $d_1(t)$, remain unchanged. For notation consistency, they are presented by X_1, $F_1(X_1)$ and $D_1(t)$, respectively. By using the above transformation and Remark 1, system (1) converts to

$$\begin{cases} \dot{X}_1 = g' X_2 + F_1(X_1) + D_1(t) \\ \dot{X}_i = X_{i+1} + F_i(\underline{X}_i) + D_i(t) \ , \ 2 \le i \le n-1 \\ \dot{X}_n = H(X_n) u(t-\tau) + F_n(\underline{X}_n) + D_n(t) \\ y = X_1 \end{cases} \tag{4}$$

where $g' = \prod_{j=1}^{n} g_j$ is defined as a new control coefficient. The above transformed dynamical system has only one unknown control coefficient which can be handled by a Nussbaum function. To design an adaptive NN controller, the following assumptions are made.

Assumption 1. The desired reference signal, y_r, and its derivative are known and bounded.

Assumption 2. There are known positive parameters $\overline{g}_i, 1 \le i \le n$ such that inequalities $0 < |g_i| \le \overline{g}_i, 1 \le i \le n-1$ are satisfied.

Assumption 3. The positive time-varying delay, $\tau(t)$, and its derivative are known.

Assumption 4. The unknown new disturbances, $D_i(t)$, $1 \le i \le n$, are bounded and there are positive constants, D_i^*, such that $|D_i(t)| \le D_i^*$, $1 \le i \le n$.

Remark 2. Regarding the above assumptions, the following points should be considered. In most of control applications, the reference signal is known and therefore the first assumption is not restrictive. Also for the real systems, the input directions and disturbances are bounded; consequently, their upper bounds should exit. It must be also noted that for implementation of the proposed control scheme, these bounds are not required. Therefore the second and fourth assumptions are not restrictive either. Regarding the third assumption, it should be mentioned that in most process control applications, the source of delay is due to liquid transfer through a pipe. For such cases, the time delay can be calculated by dividing the pipe volume by the liquid flow rate. Therefore the fourth assumption is not restrictive for many practical control applications.

Definition 1 [13, 28]**.** To compensate the effect of the unknown control direction, consider the continuous function $N(\zeta)$, called the Nussbaum gain function, which has the following properties:

$$\begin{cases} \lim_{s\to+\infty} sup\, \frac{1}{s}\int_0^s N(\zeta)d\zeta = +\infty \\ \lim_{s\to+\infty} inf\, \frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty \end{cases} \quad (5)$$

These integral equations indicate that the Nussbaum functions have infinite frequencies of switching sign. According to [29], there are many functions that satisfy equation (5). Continuous functions $\zeta^2 sin(\zeta)$ and $\zeta^2 cos(\zeta)$ are good examples of the Nussbaum gain function.

Lemma 1 [11]**.** Let $V(.)$ and $\zeta(.)$ be smooth functions defined on $\left[0, t_f\right)$ with $V(t) \geq 0$, $\forall t \in \left[0, t_f\right)$ and $N(.)$ be an even smooth and continuous Nussbaum function. If the following inequality holds:

$$V(t) \leq c_0 + \int_0^t (gN(\zeta) + 1)\dot{\zeta} d\varsigma, \forall t \in \left[0, t_f\right) \quad (6)$$

where g is a nonzero constant and c_0 represents a positive constant, then $V(.)$, $\zeta(.)$ and $\int_0^t (gN(\zeta) + 1)\,\dot{\zeta} d\varsigma$ must be bounded on $\left[0, t_f\right)$.

2.2 Neural Network

In control engineering, the radial basis function neural network (RBFNN) is usually used as a tool for estimating any smooth nonlinear function over a compact set. In this paper, the following RBFNN [26] is used to approximate the continuous function $F(x): R^m \to R$ over a compact set as follows:

$$F_{NN}(Z) = W^T S(Z) \quad (7)$$

where $Z \in \Omega \subset R^m$ and $W = [w_1, w_2, \dots, w_r]^T \in R^r$ are the inputs of NN and the weight vector, respectively. Parameter $r > 1$ denotes the number of the NN nodes. The elements of the regressor vector $S(Z) = [s_1(Z), s_2(Z), \dots, s_r(Z)]^T$ denoted by $s_i(Z)$, are usually chosen to be Gaussian functions in the following form:

$$s_i(Z) = exp\left[\frac{-(Z-\overline{\mu}_i)^T (Z-\overline{\mu}_i)}{\sigma_i^2}\right], \ 1 \leq i \leq r \quad (8)$$

where $\overline{\mu}_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{im}]^T$ are the centers of the receptive fields and $\sigma_i, 1 \leq i \leq r$ are the width of Gaussian functions. It has been proved that the RBFNN is an approximator over a compact set $\Omega \subset R^m$ with any degree of accuracy. The function approximation can be written as

$$F(Z) = W^{*T}S(Z) + \varepsilon(Z), \forall Z \in \Omega \tag{9}$$

where $\varepsilon(Z)$ is the NN approximation error and W^* is given by

$$W^* \triangleq arg\, min_{W \in R^r}\{sup_{Z \epsilon \Omega}|F(Z) - W^T S(Z)|\} \tag{10}$$

Lemma 2. There exists an ideal constant vector W^*such that $|\varepsilon| \leq \varepsilon^*$ for all $Z \in \Omega$ [19].

The RBFNN given by (9) is used to approximate the following nonlinear functions:

$$\begin{cases} \bar{F}_1(\underline{X}_2) = (g' - 1)X_2 + F_1(X_1) = W_1^{*T}S_1(\underline{X}_2) + \varepsilon_1(\underline{X}_2) \\ F_i(\underline{X}_i) = W_i^{*T}S_i(\underline{X}_i) + \varepsilon_i(\underline{X}_i), \qquad 2 \leq i \leq n \\ H(\underline{X}_n) = W_H^{*T}S_H(\underline{X}_n) + \varepsilon_H(\underline{X}_n) \end{cases} \tag{11}$$

Since $X_i, 2 \leq i \leq n$ are unmeasured, equation (11) is not applicable for approximating the nonlinear functions $\bar{F}_1(.)$, $H(.)$, $F_i(.), 2 \leq i \leq n$. To solve this problem, the following approximations are used:

$$\begin{cases} (g' - 1)X_2 + F_1(X_1) = \hat{\bar{F}}_1(\hat{\underline{X}}_2) + \Delta F_1 \\ F_i(\underline{X}_i) = \hat{F}_i(\hat{\underline{X}}_i) + \Delta F_i, \qquad 2 \leq i \leq n \\ H(\underline{X}_n)u(t - \tau) = \hat{H}(\hat{\underline{X}}_n)u(t - \tau) + \Delta H \end{cases} \tag{12}$$

where $\Delta F_i, 1 \leq i \leq n$ and ΔH are defined as

$$\begin{cases} \Delta F_1 = \bar{F}_1(\underline{X}_2) - \hat{\bar{F}}_1(\hat{\underline{X}}_2) \\ \Delta F_i = F_i(\underline{X}_i) - \hat{F}_i(\hat{\underline{X}}_i), \; 2 \leq i \leq n \\ \Delta H = (H(\underline{X}_n) - \hat{H}(\hat{\underline{X}}_n))u(t - \tau(t)) \end{cases} \tag{13}$$

and

$$\begin{cases} \hat{\bar{F}}_1(\hat{\underline{X}}_2) = W_1^T S_1(\hat{\underline{X}}_2) \\ \hat{F}_i(\hat{\underline{X}}_i) = W_i^T S_i(\hat{\underline{X}}_i), \qquad 2 \leq i \leq n \\ \hat{H}(\hat{\underline{X}}_n) = W_H^T S_H(\hat{\underline{X}}_n) \end{cases} \tag{14}$$

Applying equations (11)-(14), the unknown smooth functions $\overline{F}_1(.)$, $F_i(.)$ and $H(.), 2 \leq i \leq n$ can be estimated. Utilizing (11)-(14), the transformed system (4) can be rewritten as

$$\begin{cases} \dot{X}_1 = X_2 + \widehat{\overline{F}}_1(\underline{\hat{X}}_2) + W_1^{*T} S_1(\underline{X}_2) + \varepsilon_1(\underline{X}_2) - W_1^T S_1(\underline{\hat{X}}_2) + D_1(t) \\ \dot{X}_i = X_{i+1} + \hat{F}_i(\underline{\hat{X}}_i) + W_i^{*T} S_i(\underline{X}_i) + \varepsilon_i(\underline{X}_i) - W_i^T S_i(\underline{\hat{X}}_i) + D_i(t), 2 \leq i \leq n-1 \\ \dot{X}_n = \left(W_H^{*T} S_H(\underline{X}_n) + \varepsilon_H(\underline{X}_n)\right) u(t-\tau) + \hat{F}_n(\underline{\hat{X}}_n) + W_n^{*T} S_n(\underline{X}_n) \\ \quad -W_n^T S_n(\underline{\hat{X}}_n) + \hat{H}(\underline{\hat{X}}_n) u(t-\tau) - W_H^T S_H(\underline{\hat{X}}_n) u(t-\tau) + \varepsilon_n(\underline{X}_n) + D_n(t) \\ y = X_1 \end{cases} \quad \textbf{Disp}$$

The above form of the system will be used to obtain the error dynamics of the adaptive NN observer.

3. Adaptive NN observer design

As mentioned before, the state variables $X_i, 2 \leq i \leq n$ of system (4) are unavailable. In this section, an adaptive NN observer is designed to estimate the states of system (4). The structure of the adaptive NN observer is given below

$$\begin{cases} \dot{\hat{X}}_1 = \hat{X}_2 + \hat{\overline{F}}_1(\underline{\hat{X}}_2) + k_1(y - \hat{X}_1) \\ \dot{\hat{X}}_\mathrm{i} = \hat{X}_{\mathrm{i}+1} + \hat{F}_\mathrm{i}(\underline{\hat{X}}_\mathrm{i}) + k_\mathrm{i}(y - \hat{X}_1), \quad 2 \leq \mathrm{i} \leq n-1 \\ \dot{\hat{X}}_n = \hat{H}(\underline{\hat{X}}_n) u(t-\tau) + \hat{F}_n(\underline{\hat{X}}_n) + k_n(y - \hat{X}_1) \end{cases} \tag{16}$$

where $\underline{\hat{X}}_i = [\hat{X}_1, \hat{X}_2, \ldots, \hat{X}_i]^T, 1 \leq i \leq n$ are the estimated states vectors. $\hat{F}_i(\underline{\hat{X}}_i), 1 \leq i \leq n$ and $\hat{H}(\underline{\hat{X}}_n)$ are defined by (14). The positive gains, $k_i, 1 \leq i \leq n$ are chosen such that the polynomial $P(S) = S^n + k_1 S^{n-1} + \cdots + k_{n-1} S + k_n$ is Hurwitz. Defining the state observation error vector as $e = X - \hat{X} = [e_1, e_2, \ldots, e_n]^T$, and substituting equations (14)-(16) into time derivate of the observer error, results in the observation error dynamics as given below

$$\begin{cases} \dot{e_1} = e_2 + W_1^{*T} S_1(\underline{X}_2) + \varepsilon_1(\underline{X}_2) - W_1^T S_1(\underline{\hat{X}}_2) - k_1 e_1 + D_1(t) \\ \dot{e_\mathrm{i}} = e_{\mathrm{i}+1} + W_\mathrm{i}^{*T} S_\mathrm{i}(\underline{X}_\mathrm{i}) + \varepsilon_\mathrm{i}(\underline{X}_\mathrm{i}) - W_\mathrm{i}^T S_\mathrm{i}(\underline{\hat{X}}_\mathrm{i}) - k_\mathrm{i}\, e_1 + D_\mathrm{i}(t), 2 \le \mathrm{i} \le n-1 \\ \dot{e_n} = \left(W_H^{*T} S_H(\underline{X}_n) - W_H^T S_H(\underline{\hat{X}}_n)\right) u(t-\tau) + W_n^{*T} S_n(\underline{X}_n) - W_n^T S_n(\underline{\hat{X}}_n) \\ \quad + \varepsilon_n(\underline{X}_n) u(t-\tau) - k_n e_1 + D_n(t) \end{cases} \tag{17}$$

The above equation can be expressed in the matrix form. By defining $\widetilde{W} = W^* - W$, and $\Upsilon_i(\underline{X}_i, \underline{\hat{X}}_i) = S_i(\underline{X}_i) - S_i(\underline{\hat{X}}_i)$, the observation error dynamic can be written as

$$\dot{e} = Ae + W^{*T}\Upsilon(\underline{X}, \underline{\hat{X}}) + W_\Xi^{*T}\Theta\left(\underline{X}, \underline{\hat{X}}, u(t-\tau)\right) + \widetilde{W}^T S(\underline{\hat{X}}) + \Gamma + \varepsilon + \Lambda + D \tag{18}$$

where

$$A = \begin{bmatrix} -k_1 & & \\ \vdots & I_{n-1} & \\ -k_n & \cdots & \end{bmatrix}_{n\times n}, \varepsilon = \left[\varepsilon_1(\underline{X}_2), \dots, \varepsilon_n(\underline{X}_n)\right]^T, \; D = \left[D_1(\mathrm{t}), \dots, D_n(\mathrm{t})\right]^T$$

$$\Gamma = \left[0, \dots, \widetilde{W}_H^T S_H(\underline{\hat{X}}_n) u(t-\tau)\right]_{1\times n}^T, \Lambda = \left[0, \dots, \varepsilon_H(\underline{X}_n) u(t-\tau)\right]_{1\times n}^T$$

$$W^{*T}\Upsilon(\underline{X}, \underline{\hat{X}}) = \left[W_1^{*T}\Upsilon_1(\underline{X}_2, \underline{\hat{X}}_2), W_2^{*T}\Upsilon_2(\underline{X}_2, \underline{\hat{X}}_2), \dots, W_n^{*T}\Upsilon_n(\underline{X}_n, \underline{\hat{X}}_n)\right]^T$$

$$\widetilde{W}^T S(\underline{\hat{X}}) = \left[\widetilde{W}_1^T S_1(\underline{\hat{X}}_2), \widetilde{W}_2^T S_2(\underline{\hat{X}}_2), \dots, \widetilde{W}_n^T S_n(\underline{\hat{X}}_n)\right]^T$$

$$W_\Xi^{*T}\Theta\left(\underline{X}_n, \underline{\hat{X}}_n, u(t-\tau)\right) = \left[0, \dots, W_H^{*T}\Upsilon_H(\underline{X}_n, \underline{\hat{X}}_n) u(t-\tau)\right]_{1\times n}^T.$$

It should be noted that to derive the observation error dynamic (18), the following equalities have been used:

$$\begin{cases} W_1^{*T} S_1(\underline{X}_2) - W_1^T S_1(\underline{\hat{X}}_2) = W_1^{*T}\Upsilon_1(\underline{X}_2, \underline{\hat{X}}_2) + \widetilde{W}_1^T S_1(\underline{\hat{X}}_2) \\ W_i^{*T} S_i(\underline{X}_i) - W_i^T S_i(\underline{\hat{X}}_i) = W_i^{*T}\Upsilon_i(\underline{X}_i, \underline{\hat{X}}_i) + \widetilde{W}_i^T S_i(\underline{\hat{X}}_i), 2 \le i \le n \\ W_H^{*T} S_H(\underline{X}_n) - W_H^T S_H(\underline{\hat{X}}_n) = W_H^{*T}\Upsilon_H(\underline{X}_n, \underline{\hat{X}}_n) + \widetilde{W}_H^T S_H(\underline{\hat{X}}_n) \end{cases} \tag{19}$$

Remark 3. Using the approach proposed in [1] and properties of the Gaussian function, we have the following inequalities which will be used later in the stability analysis of the observer:

$$\begin{cases} \Upsilon_i^T(\underline{X}_i,\underline{\hat{X}}_i)\Upsilon_i(\underline{X}_i,\underline{\hat{X}}_i) < r_i, & 1 \le i \le n \\ \Upsilon_H^T(\underline{X}_n,\underline{\hat{X}}_n)\Upsilon_H(\underline{X}_n,\underline{\hat{X}}_n) < r_H \end{cases} \tag{20}$$

where $r_i, 1 \le i \le n$ and r_H are numbers of the network neurons in approximating the ith nonlinear function $F_i(.)$ and $H(.)$, respectively. The above inequalities imply that each element of $\Upsilon_i(\underline{X}_i,\underline{\hat{X}}_i), 1 \le i \le n$ and $\Upsilon_H(\underline{X}_n,\underline{\hat{X}}_n)$ are bounded and their maximum values are one. It should be noted that inequality (20) is also valid for regressor vectors $S_i(.), 1 \le i \le n$ and $S_H(.)$. By using inequality (20) and Remark 3, the Lipschitz condition can be relaxed in the controller design.

To analyze the stability of the error dynamic (18), consider the following Lyapunov function:

$$V_0 = e^T P e \tag{21}$$

where $P > 0$ is a symmetric matrix. By substituting (18) into time derivative of the above Lyapunov function one gets

$$\dot{V}_0 = e^T(A^T P + PA)e + 2e^T P\left(\tilde{W}^T S(\underline{\hat{X}}) + W_\Xi^{*T}\Theta(\underline{X},\underline{\hat{X}},u(t-\tau)) + W^{*T}\Upsilon(\underline{X},\underline{\hat{X}})\right) + 2e^T P(\Gamma + \varepsilon + \Lambda + D) \tag{22}$$

To proceed the stability analysis of the adaptive NN observer, the second and third terms in right hand side of (22) need to be separated from the estimation error vector

$e = [e_1, e_2, \dots, e_n]^T$. Using Remark 3, Lemma 2, Assumption 4 and Young's inequality, the following inequality can be obtained:

$$2e^T P \left(W^{*T}\Upsilon\left(\underline{X}, \underline{\hat{X}}\right) + W_{\Xi}^{*T}\Theta\left(\underline{X}, \underline{\hat{X}}, u(t-\tau)\right) + \widetilde{W}^T S\left(\underline{\hat{X}}\right)\right) + 2e^T P(\Gamma + \varepsilon + \Lambda + D) \leq 3e^T PPe\left(\frac{4}{3} + u_m\right) + u_m r_H\left(W_H^{*T}W_H^* + \widetilde{W}_H^T\widetilde{W}_H\right) + \sum_{j=1}^{n} r_j\left(W_j^{*T}W_j^* + \widetilde{W}_j^T\widetilde{W}_j\right) + \|\varepsilon^*\|^2 + u_m\|\varepsilon_H^*\|^2 + \|D^*\|^2 \tag{23}$$

where vectors D^* and ε^* are defined as $D^* = [D_1^*, \dots, D_n^*]^T$ and $\varepsilon^* = [\varepsilon_1^*, \dots, \varepsilon_n^*]^T$. Using inequality (23) in (22) yields

$$\dot{V}_0 \leq e^T\left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right)PP\right)e + R_1 \tag{24}$$

where R_1 is defined as

$$R_1 = \sum_{j=1}^{n} r_j(W_j^{*T}W_j^* + \widetilde{W}_j^T\widetilde{W}_j) + u_m r_H\left(W_H^{*T}W_H^* + \widetilde{W}_H^T\widetilde{W}_H\right) + \|\varepsilon^*\|^2 + u_m\|\varepsilon_H^*\|^2 + \|D^*\|^2 \tag{25}$$

Inequality (24) will be used in the stability analysis of the closed-loop system in the presence of observer dynamics.

4. Adaptive controller design

In this section, the procedure of designing an adaptive output feedback NN controller based on the backstepping technique for system (4) is presented. Designing the adaptive NN controller includes n-steps. The following change of coordinate is introduced:

$$\begin{cases} z_1 = y - y_r \\ z_i = \hat{X}_i - \omega_i\,, \qquad\qquad 2 \le i \le n-1 \\ z_n = \hat{X}_n - \omega_n - \lambda + \hat{H}\left(\underline{\hat{X}}_n\right)\int_{t-\tau(t)}^{t} u(s)ds \end{cases} \tag{26}$$

where $z_i, 1 \le i \le n$ and y_r are the control surfaces and the desired trajectory, respectively. Variable λ is the output of a first order differential equation which will be defined in the final step. The integral term appeared in the last control surface has been considered to compensate the effect of input delay. Variables $\omega_i, 2 \le i \le n$ are the outputs of the first order filters with adjustable time constants $\pi_i, 2 \le i \le n$ as given below

$$\dot{\omega}_i\pi_i + \omega_i = \alpha_{i-1}\,, \omega_i(0) = \alpha_{i-1}(0), \quad 2 \le i \le n \tag{27}$$

Using the above filters, computational burden in calculating the virtual control signals, $\alpha_j, 1 \le j \le n-1$ and input signal, ν, will be decreased compared to the load required for the traditional backstepping technique. To proceed the DSC controller design, the variables $L_i, 2 \le i \le n$ are defined as

$$L_{i-1} = \omega_i - \alpha_{i-1}, \quad 2 \le i \le n \tag{28}$$

Using (27), the following equalities for time derivatives of L_{i-1} can be obtained:

$$\dot{L}_{i-1} = \dot{\omega}_i - \dot{\alpha}_{i-1} = -\frac{L_{i-1}}{\pi_i} + Q_{i-1}, \quad 2 \le i \le n \tag{29}$$

According to [39], $Q_{i-1} = \dot{\alpha}_{i-1}, 2 \le i \le n$ are continuous functions. Equations (28) and (29) are used in the stability analysis of the closed-loop system and finding the appropriate intervals for time constants $\pi_i, 2 \le i \le n$.

Step 1: By taking time derivative of the first control surface (26) and substituting for the first state of system (4), one has

$$\dot{z}_1 = g' X_2 + F_1(X_1) + D_1(t) - \dot{y}_r \tag{30}$$

Because the nonlinear function $F_1(X_1)$ is unknown, it is approximated with a NN as given below

$$F_1(X_1) = W_{s1}^{*T} S_{s1}(X_1) + \varepsilon_{s1}(X_1) \tag{31}$$

Using (26), (28) and (31), $\dot{z}_1$ can be written as

$$\dot{z}_1 = g'(z_2 + \alpha_1 + L_1 + e_2) + W_{s1}^{*T} S_{s1}(X_1) + \varepsilon_{s1}(X_1) + D_1 - \dot{y}_r \tag{32}$$

where $e_2 = X_2 - \hat{X}_2$.

The following Lyapunov function has been considered for the first step:

$$V_1 = V_0 + \frac{1}{2} z_1^2 + \frac{1}{2} L_1^2 + \frac{1}{2\gamma_1} \widetilde{W}_1^T \widetilde{W}_1 + \frac{1}{2\gamma_{s1}} \widetilde{W}_{s1}^T \widetilde{W}_{s1} \tag{33}$$

where γ_{s1} and γ_1 are the positive design constants. By substituting (24), (29) and (32) into time derivative of the above Lyapunov function and adding and subtracting term $\widetilde{W}_1^T S_1(\underline{\hat{X}}_2) z_1$ we obtain

$$\dot{V}_1 \leq e^T \left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP \right) e - \frac{1}{\gamma_1} \widetilde{W}_1^T \left(\dot{W}_1 - \gamma_1 S_1(\underline{\hat{X}}_2) z_1 \right) +$$
$$z_1 \left(g'(z_2 + \alpha_1 + L_1) - \dot{y}_r + \left(W_{s1}^T + \widetilde{W}_{s1}^T \right) S_{s1}(X_1) - \widetilde{W}_1^T S_1(\underline{\hat{X}}_2) \right) -$$
$$\frac{1}{\gamma_{s1}} \widetilde{W}_{s1}^T \dot{W}_{S1} - L_1 \left(\frac{L_1}{\pi_2} - Q_1 \right) + z_1 (g' e_2 + \varepsilon_{s1}(X_1) + D_1) + R_1 \tag{34}$$

Using Assumption 2, Remark 3, Lemma 2, (20) and Young's inequality, the following inequalities are obtained:

$$z_1 z_2 g' \leq (z_1 z_2)^2 + \frac{\left(\overline{g}'\right)^2}{4} \tag{35}$$

$$-z_1 \widetilde{W}_1^T S_1\left(\underline{\hat{X}}_2\right) \leq z_1^2 + r_1 \frac{\widetilde{W}_1^T \widetilde{W}_1}{4} \tag{36}$$

$$z_1(g' e_2 + \varepsilon_{s1}(X_1) + D_1) + L_1(Q_1 + z_1) \leq 4z_1^2 + \frac{\overline{g}'^2}{4} e^T e + \frac{\left\|\varepsilon_{S1}^*\right\|^2}{4} + \frac{\|D^*\|^2}{4} + \frac{L_1^2}{2} + Q_1^2 \tag{37}$$

Substituting inequalities (35)-(37) into (34) yields

$$\dot{V}_1 \leq e^T \left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I\right) e - L_1^2 \left(\frac{1}{\pi_2} - \frac{1}{2}\right) + z_1(5z_1 + g'\alpha_1 + W_{S1}^T S_{s1}(X_1) - \dot{y}_r) + (z_1 z_2)^2 - \frac{1}{\gamma_1} \widetilde{W}_1^T\left(\dot{W}_1 - \gamma_1 S_1\left(\underline{\hat{X}}_2\right) z_1\right) - \frac{1}{\gamma_{s1}} \widetilde{W}_{S1}^T\left(\dot{W}_{S1} - \gamma_{s1} S_{s1}(X_1) z_1\right) + Q_1^2 + \frac{\overline{g}'^2}{4} + r_1 \frac{\widetilde{W}_1^T \widetilde{W}_1}{4} + \frac{\left\|\varepsilon_{S1}^*\right\|^2}{4} + \frac{\|D^*\|^2}{4} + R_1 \tag{38}$$

where $\overline{g}' = \prod_{j=1}^{n} \overline{g}_j$. Based on Assumption 2, $\overline{g}'$ is positive and known. It should be noted that to derive (38), $\dot{\widetilde{W}}_1 = -\dot{W}_1$ and $\dot{\widetilde{W}}_{s1} = -\dot{W}_{s1}$ have been used. For the first step, the following virtual control signal α_1, tuning function ζ, and adaptation laws for W_1, W_{S1} are proposed:

$$\alpha_1 = N(\zeta)(C_1 z_1 + 5z_1 + W_{S1}^T S_{s1}(X_1) - \dot{y}_r) \tag{39}$$

$$\dot{\zeta} = \frac{z_1}{K}(C_1 z_1 + 5z_1 + W_{S1}^T S_{s1}(X_1) - \dot{y}_r) \tag{40}$$

$$\dot{W}_1 = \gamma_1 z_1 S_1\left(\underline{\hat{X}}_2\right) - \sigma(W_1 - W_1^0) \tag{41}$$

$$\dot{W}_{s1} = \gamma_{s1} z_1 S_{s1}(X_1) - \sigma(W_{s1} - W_{s1}^0) \tag{42}$$

where terms $\sigma(W_i - W_i^0), i = 1, s_1$ are called sigma-modification which make the adaptive laws robust in the presence of NN approximation error [3, 10]. C_1, K and σ are positive design constants. W_{s1}^0 and W_1^0 are strictly positive design vectors. Substituting the virtual control signal (39), (40), and adaptation laws (41), (42) into time derivative of V_1 gives

$$\dot{V}_1 \leq e^T \left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I \right) e + K(g' N(\zeta) + 1)\dot{\zeta} -$$

$$C_1 z_1^2 + (z_1 z_2)^2 + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{S1}^T (W_{s1} - W_{s1}^0) + \frac{\sigma}{\gamma_1} \widetilde{W}_1^T (W_1 - W_1^0) -$$

$$L_1^2 \left(\frac{1}{\pi_2} - \frac{1}{2}\right) + r_1 \frac{\widetilde{W}_1^T \widetilde{W}_1}{4} + Q_1^2 + R_2 \tag{43}$$

where

$$R_2 = R_1 + \frac{\|\varepsilon_{S1}^*\|^2}{4} + \frac{\|D^*\|^2}{4} + \frac{\overline{g}'^2}{4} \tag{44}$$

Step 2: Using the adaptive NN observer (16), (26) and (45), time derivative of the second control surface can be written as

$$\dot{z_2} = z_3 + \alpha_2 + L_2 + W_2^T S_2(\underline{\hat{X}}_2) + k_2(y - \hat{X}_1) - \dot{\omega}_2 \tag{46}$$

For the second step, the following Lyapunov function has been chosen:

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2} L_2^2 + \frac{1}{2\gamma_2} \widetilde{W}_2^T \widetilde{W}_2 \tag{47}$$

where γ_2 is a positive design parameter. Substituting (29), (43) and $\dot{L}_2 = -\frac{L_2}{\pi_3} + Q_2$ into time derivative of V_2 gives

$$\dot{V}_2 \leq e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + K(g'N(\zeta) + 1)\dot{\zeta} - C_1z_1^2 - \frac{1}{\gamma_2}\tilde{W}_2^T\left(\dot{W}_2 - \gamma_2 S_2\left(\underline{\hat{X}}_2\right)z_2\right) + z_2\left(z_2z_1^2 + z_3 + \alpha_2 + L_2 + W_2^TS_2\left(\underline{\hat{X}}_2\right) - \tilde{W}_2^TS_2\left(\underline{\hat{X}}_2\right) - k_2\left(y - \hat{X}_1\right) - \dot{\omega}_2\right) + \frac{\sigma}{\gamma_1}\tilde{W}_1^T(W_1 - W_1^0) + \frac{\sigma}{\gamma_{s1}}\tilde{W}_{S1}^T(W_{s1} - W_{s1}^0) - L_1^2\left(\frac{1}{\pi_2} - \frac{1}{2}\right) + L_2\left(\frac{-L_2}{\pi_3} + Q_2\right) + r_1\frac{\tilde{W}_1^T\tilde{W}_1}{4} + Q_1^2 + R_2 \tag{48}$$

Note that to derive the inequality (48), term $\tilde{W}_2^TS_2\left(\underline{\hat{X}}_2\right)z_2$ is added and subtracted. Using Young's inequality and Remark 3 we have

$$z_2L_2 - z_2\tilde{W}_2^TS_2\left(\underline{\hat{X}}_2\right) + L_2Q_2 \leq 2z_2^2 + \frac{L_2^2}{2} + r_2\frac{\tilde{W}_2^T\tilde{W}_2}{4} + Q_2^2 \tag{49}$$

Substituting (49) into (48) yields

$$\dot{V}_2 \leq e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + K(g'N(\zeta) + 1)\dot{\zeta} + -C_1z_1^2 + z_2\left(z_2z_1^2 + 2z_2 + \alpha_2 + W_2^TS_2\left(\underline{\hat{X}}_2\right) + k_2\left(y - \hat{X}_1\right) - \dot{\omega}_2\right) + z_2z_3 - \frac{1}{\gamma_2}\tilde{W}_2^T\left(\dot{W}_2 - \gamma_2S_2\left(\underline{\hat{X}}_2\right)z_2\right) + \frac{\sigma}{\gamma_1}\tilde{W}_1^T(W_1 - W_1^0) + \frac{\sigma}{\gamma_{s1}}\tilde{W}_{S1}^T(W_{s1} - W_{s1}^0) - \sum_{j=1}^2 L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{j=1}^2 r_j\frac{\tilde{W}_j^T\tilde{W}_j}{4} + \sum_{j=1}^2 Q_j^2 + R_2 \tag{50}$$

Consider the following second virtual control signal and adaptation law as

$$\alpha_2 = -C_2z_2 - 2z_2 - z_2z_1^2 - W_2^TS_2\left(\underline{\hat{X}}_2\right) - k_2\left(y - \hat{X}_1\right) + \dot{\omega}_2 \tag{51}$$

$$\dot{W}_2 = \gamma_2z_2S_2\left(\underline{\hat{X}}_2\right) - \sigma(W_2 - W_2^0) \tag{52}$$

where C_2 and σ are the positive design constants and W_2^0 is a strictly positive design vector. Using virtual control (51) and adaptation law (52), (50) can be rewritten as

$$\dot{V}_2 \le e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e \ + \ K(g'N(\zeta) + 1)\dot{\zeta} - \sum_{j=1}^{2} L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) - \sum_{k=1}^{2} C_k z_k^2 + z_2 z_3 + \frac{\sigma}{\gamma_{s1}}\widetilde{W}_{s1}^T\left(W_{s1} - W_{s1}^0\right) + \sum_{k=1}^{2} \frac{\sigma}{\gamma_k}\widetilde{W}_k^T\left(W_k \ - W_K^0\right) + \sum_{k=1}^{2} r_k \frac{\widetilde{W}_k^T\widetilde{W}_k}{4} + \sum_{j=1}^{2} Q_j^2 + R_2 \quad (53)$$

Step $\boldsymbol{i}$, $3 \le i \le n-1$: Using the adaptive NN observer (16), control surfaces (26) and $L_i = \omega_{i+1} - \alpha_i$, time derivative of the ith control surface can be written as:

$$z_i = \alpha_i + L_i + z_{i+1} + W_i^T S_i\left(\underline{\hat{X}}_i\right) + k_i\left(y - \hat{X}_1\right) - \dot{\omega}_i \quad (54)$$

For this step, consider the following Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}L_i^2 + \frac{1}{2\gamma_i}\widetilde{W}_i^T\widetilde{W}_i \quad (55)$$

where γ_i is a positive design constant. From the previous step, we have the following inequality for $\dot{V}_{i-1}$:

$$\dot{V}_{i-1} \le e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + \ K(g'N(\zeta) + 1)\dot{\zeta} - \sum_{k=1}^{i-1} C_k z_k^2 + \frac{\sigma}{\gamma_{s1}}\widetilde{W}_{s1}^T\left(W_{s1} - W_{s1}^0\right) + \sum_{k=1}^{i-1} \frac{\sigma}{\gamma_k}\widetilde{W}_k^T\left(W_k \ - W_K^0\right) - \sum_{j=1}^{i-1} L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + z_i z_{i-1} + \sum_{k=1}^{i-1} r_k \frac{\widetilde{W}_k^T\widetilde{W}_k}{4} + \sum_{j=1}^{i-1} Q_j^2 + R_2 \quad (56)$$

Substituting $\dot{V}_{i-1}$, (29) and (54) into the time derivative of V_i and adding and subtracting term $\widetilde{W}_i^T S_i\left(\underline{\hat{X}_i}\right) z_i$ yields

$$\dot{V}_i \le e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + \ K(g'N(\zeta) + 1)\dot{\zeta} + z_i z_{i+1} - \sum_{k=1}^{i-1} C_k z_k^2 - \sum_{j=1}^{i} L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) - \frac{1}{\gamma_i}\widetilde{W}_i^T\left(\dot{W}_i - \gamma_i z_i S_i\left(\underline{\hat{X}_i}\right)\right) + \frac{\sigma}{\gamma_{s1}}\widetilde{W}_{s1}^T\left(W_{s1} -\right.$$

$$W_{s1}^0) + \sum_{k=1}^{i-1} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T (W_k \ - W_K^0) + L_i \left(-\frac{L_i}{\pi_{i+1}} + Q_i\right) + z_i \left(z_{i-1} + L_i + \alpha_i + \right.$$

$$\left. W_i^T S_i(\underline{\hat{X}}_i) + k_i(y - \hat{X}_1) - \dot{\omega}_i - \widetilde{W}_i^T S_i(\underline{\hat{X}}_i)\right) + \sum_{k=1}^{i} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + \sum_{j=1}^{i} Q_j^2 + R_2$$

(57)

Utilizing Remark 3 and Young's inequality, the following inequality can obtained:

$$L_i Q_i + z_i L_i - z_i \widetilde{W}_i^T S_i(\underline{\hat{X}}_i) \leq 2z_i^2 + \frac{L_i^2}{2} + r_i \frac{\widetilde{W}_i^T \widetilde{W}_i}{4} + Q_i^2 \tag{58}$$

Substituting the above inequality into (57) gives

$$\dot{V}_i \leq e^T \left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I\right) e + \ K(g' N(\zeta) + 1)\dot{\zeta} +$$

$$\sum_{j=1}^{i} Q_j^2 - \sum_{k=1}^{i-1} C_k z_k^2 + \sum_{k=1}^{i-1} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T (W_k \ - W_K^0) + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{s1}^T (W_{s1} - W_{s1}^0) -$$

$$\frac{1}{\gamma_i} \widetilde{W}_i^T \left(\dot{W}_i - \gamma_i z_i S_i(\underline{\hat{X}}_i)\right) + z_i\big(z_{i-1} + 2z_i + \alpha_i + W_i^T S_i(\underline{\hat{X}}_i) + k_i(y - \hat{X}_1) -$$

$$\dot{\omega}_i\big) + z_i z_{i+1} - \sum_{j=1}^{i} L_j^2 \left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{k=1}^{i} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + R_2 \tag{59}$$

The following virtual control signal and adaptation law are proposed:

$$\alpha_i = -C_i z_i - z_{i-1} - 2z_i - W_i^T S_i(\underline{\hat{X}}_i) - k_i(y - \hat{X}_1) + \dot{\omega}_i \tag{60}$$

$$\dot{W}_i = \gamma_i z_i S_i(\underline{\hat{X}}_i) - \sigma(W_i - W_i^0) \tag{61}$$

where $C_i, 3 \leq i \leq n-1$ and σ are positive design constants and $W_i^0, 3 \leq i \leq n-1$ are positive design vectors. By using virtual control (60) and adaptation law (61), $\dot{V}_i$ can be rewritten as

$$\dot{V}_i \leq e^T \left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I\right) e + \ K(g' N(\zeta) + 1)\dot{\zeta} +$$

$$\sum_{k=1}^{i} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T \left(W_k \ - W_K^0\right) + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{s1}^T \left(W_{s1} - W_{s1}^0\right) - \sum_{k=1}^{i} C_k z_k^2 + z_i z_{i+1} - \tag{62}$$

$$\sum_{j=1}^{i} L_j^2 \left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{k=1}^{i} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + \sum_{j=1}^{i} Q_j^2 + R_2$$

Step n: Similar to the approach proposed in [1], to handle the input nonlinearities and design a nonsingular controller, the following first order differential equation for the last step is considered:

$$\dot{\lambda} = \frac{\vartheta}{\hat{H}(\underline{\hat{X}}_n)^2 + \vartheta} \left(C_n z_n + W_S^T S_S(\psi) + z_{n-1} + 3z_n + C_H(t) - \dot{\omega}_n\right) -$$

$$\beta\left|\hat{H}\left(\underline{\hat{X}}_n\right)\right|\lambda + \hat{H}\left(\underline{\hat{X}}_n\right)\left(u(t) - \nu(t)\right) \tag{63}$$

where $C_H(t)$ is given by

$$C_H(t) = \widehat{F_n}\left(\underline{\hat{X}}_n\right) + k_n e_1 + \left(\dot{W}_H^T S_H\left(\underline{\hat{X}}_n\right)\right) \int_{t-\tau(t)}^{t} u(s) ds \tag{64}$$

β, ϑ and C_n are positive design constants. Term $W_S^T S_S(\psi)$ in differential equation (63) is used to estimate the integral term appeared in time derivative of the nth control surface. By substituting the last state of observer (16) and differential equation (63) into time derivative of z_n, the following equation can be obtained:

$$\dot{z}_n = \hat{H}\left(\underline{\hat{X}}_n\right)\nu + W_S^T S_S(\psi) + \varepsilon_S(\psi) + C_H(t) + \beta\left|\hat{H}\left(\underline{\hat{X}}_n\right)\right|\lambda +$$

$$\hat{H}\left(\underline{\hat{X}}_n\right)\dot{\tau}(t)u(t-\tau) - \frac{\vartheta}{\hat{H}(\underline{\hat{X}}_n)^2 + \vartheta} \left(C_n z_n + W_S^T S_S(\psi) + z_{n-1} + 3z_n +\right.$$

$$\left. C_H(t) - \dot{\omega}_n\right) - \dot{\omega}_n$$

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where $\dot{\tau}(t)$ is time derivative of $\tau(t)$. Similar to the approach proposed in [12, 24, 43], in order to avoid the computational burden of complex terms arising from time derivative of $S_H\left(\underline{\hat{X}}_n\right)$ in (65), the following NN approximation is used:

$$\left(\sum_{k=1}^{n} W_H^T \frac{\partial S_H(\hat{\underline{X}}_n)}{\partial \hat{\underline{X}}_k} \dot{\hat{\underline{X}}}_k\right) \int_{t-\tau(t)}^{t} u(s)ds = W_S^{*T} S_S(\psi) + \varepsilon_S(\psi) \tag{66}$$

where $\psi = \left[W_H, \hat{\underline{X}}_n, y, \int_{t-\tau(t)}^{t} u(s)ds\right]^T$ is the input vector to construct the NN regressors. For the last step, the following Lyapunov function is proposed:

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2\gamma_n} \widetilde{W}_n^T \widetilde{W}_n + \frac{1}{2\gamma_H} \widetilde{W}_H^T \widetilde{W}_H + \frac{1}{2\gamma_S} \widetilde{W}_S^T \widetilde{W}_S \tag{67}$$

where γ_n, γ_H and γ_S are positive design constants. Using (65) and time derivative of V_{n-1} (obtained from (62) for $i = n-1$) in time derivative of (67) and adding and subtracting term $z_n\left(\widetilde{W}_n^T S_n(\hat{\underline{X}}_n) + \widetilde{W}_H^T S_H(\hat{\underline{X}}_n)\right)$, leads to the following inequality:

$$\begin{aligned}
\dot{V}_n \leq\ & e^T\left(A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I\right) e + K(g' N(\zeta) + 1)\dot{\zeta} - \\
& \sum_{j=1}^{n-1} L_j^2 \left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{j=1}^{n-1} Q_j^2 - \sum_{k=1}^{n-1} C_k z_k^2 - \frac{1}{\gamma_H} \widetilde{W}_H^T \left(\dot{W}_H - \gamma_H z_n S_H(\hat{\underline{X}}_n)\right) - \\
& \frac{1}{\gamma_n} \widetilde{W}_n^T \left(\dot{W}_n - \gamma_n z_n S_n(\hat{\underline{X}}_n)\right) + \sum_{k=1}^{n-1} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T \left(W_k - W_K^0\right) + z_n\left(\hat{H}(\underline{X}_n)\nu - \right. \\
& \left. \widetilde{W}_n^T S_n\left(\hat{\underline{X}}_n\right) - \widetilde{W}_H^T S_H\left(\hat{\underline{X}}_n\right) + W_S^T S_S(\psi) + \varepsilon_S(\psi) + z_{n-1} + C_H(t)\right) - \\
& z_n \frac{\vartheta}{\hat{H}(\underline{X}_n)^2 + \vartheta} \left(C_n z_n + W_S^T S_S(\psi) + z_{n-1} + 3z_n + C_H(t) - \dot{\omega}_n\right) + \\
& z_n\left(\hat{H}(\underline{X}_n)\dot{\tau}(t) u(t-\tau) + \beta\left|\hat{H}(\underline{X}_n)\right|\lambda - \dot{\omega}_n\right) + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{s1}^T \left(W_{s1} - W_{s1}^0\right) - \\
& \frac{1}{\gamma_S} \widetilde{W}_S^T \left(\dot{W}_S - \gamma_S z_n S_S(\psi)\right) + \sum_{k=1}^{n} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + R_2
\end{aligned} \tag{68}$$

Applying Young's inequality, Remark 3, and Lemma 2 results in

$$z_n\left(\varepsilon_S(\psi) - \widetilde{W}_n^T S_n(\hat{\underline{X}}_n) - \widetilde{W}_H^T S_H(\hat{\underline{X}}_n)\right) \leq 3z_n^2 + r_n \frac{\widetilde{W}_n^T \widetilde{W}_n}{4} + r_H \frac{\widetilde{W}_H^T \widetilde{W}_H}{4} + \frac{\|\varepsilon_S^*\|^2}{4} \tag{69}$$

Using the above inequality in (68) gives

$$\dot{V}_n \leq e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + K(g'N(\zeta) + 1)\dot{\zeta} +$$

$$\sum_{j=1}^{n-1} Q_j^2 - \sum_{k=1}^{n-1} C_k z_k^2 - \sum_{j=1}^{n-1} L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{k=1}^{n-1}\frac{\sigma}{\gamma_k}\widetilde{W}_k^T\left(W_k - W_K^0\right) +$$

$$\frac{\sigma}{\gamma_{s1}}\widetilde{W}_{s1}^T\left(W_{s1} - W_{s1}^0\right) - \frac{1}{\gamma_n}\widetilde{W}_n^T\left(\dot{W}_n - \gamma_n z_n S_n\left(\underline{\hat{X}}_n\right)\right) - \frac{1}{\gamma_H}\widetilde{W}_H^T\left(\dot{W}_H -\right.$$

$$\left.\gamma_H z_n S_H\left(\underline{\hat{X}}_n\right)\right) - \frac{1}{\gamma_S}\widetilde{W}_S^T\left(\dot{W}_S - \gamma_S z_n S_S(\psi)\right) + z_n\left(\hat{H}\left(\underline{\hat{X}}_n\right)\nu + W_S^T S_S(\psi) + z_{n-1} +\right.$$

$$\left.3z_n + C_H(t) - \dot{\omega}_n\right) + z_n\left(\hat{H}\left(\underline{\hat{X}}_n\right)\dot{\tau}(t)u(t-\tau) + \beta\left|\hat{H}\left(\underline{\hat{X}}_n\right)\right|\lambda\right) -$$

$$z_n\frac{\vartheta}{\hat{H}\left(\underline{\hat{X}}_n\right)^2 + \vartheta}\left(C_n z_n + W_S^T S_S(\psi) + z_{n-1} + 3z_n + C_H(t) - \dot{\omega}_n\right) +$$

$$\sum_{k=1}^{n} r_k\frac{\widetilde{W}_k^T\widetilde{W}_k}{4} + r_H\frac{\widetilde{W}_H^T\widetilde{W}_H}{4} + \frac{\left\|\varepsilon_S^*\right\|^2}{4} + R_2 \tag{70}$$

The following control signal and adaptation laws are proposed:

$$\nu = -\frac{\hat{H}\left(\underline{\hat{X}}_n\right)}{\hat{H}\left(\underline{\hat{X}}_n\right)^2 + \vartheta}\left(C_n z_n + W_S^T S_S(\psi) + z_{n-1} + 3z_n + C_H(t) - \dot{\omega}_n\right) - \dot{\tau}(t)u(t-\tau) - \beta\lambda sign\left(\hat{H}\left(\underline{\hat{X}}_n\right)\right) \tag{71}$$

$$\dot{W}_n = \gamma_n z_n S_n\left(\underline{\hat{X}}_n\right) - \sigma\left(W_n - W_n^0\right) \tag{72}$$

$$\dot{W}_H = \gamma_H z_n S_H\left(\underline{\hat{X}}_n\right) - \sigma\left(W_H - W_H^0\right) \tag{73}$$

$$\dot{W}_S = \gamma_S z_n S_S(\psi) - \sigma\left(W_S - W_S^0\right) \tag{74}$$

where W_n^0, W_H^0 and W_S^0 are strictly positive design vectors. Substituting (71) and (72) - (74) into (70) yields

$$\dot{V}_n \leq e^T\left(A^TP + PA + 3\left(\frac{4}{3} + u_m\right)PP + \frac{\overline{g}'^2}{4}I\right)e + K(g'N(\zeta) + 1)\dot{\zeta} +$$

$$\sum_{j=1}^{n-1} Q_j^2 - \sum_{j=1}^{n-1} L_j^2 \left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) - \sum_{k=1}^{n} C_k z_k^2 + \sum_{k=1}^{n} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T \left(W_k - W_K^0\right) + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{s1}^T \left(W_{s1} - W_{s1}^0\right) + \frac{\sigma}{\gamma_H} \widetilde{W}_H^T \left(W_H - W_H^0\right) + \frac{\sigma}{\gamma_S} \widetilde{W}_S^T \left(W_S - W_S^0\right) + \sum_{k=1}^{n} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + r_H \frac{\widetilde{W}_H^T \widetilde{W}_H}{4} + \frac{\|\varepsilon_S^*\|^2}{4} + R_2 \tag{75}$$

In order to make the closed-loop system stable, a positive and symmetric matrix $Q \in R^{n\times n}$ is selected and the following algebraic Lyapunov equation is solved to obtain a positive systematic matrix P satisfying the following inequality:

$$A^T P + PA + 3\left(\frac{4}{3} + u_m\right) PP + \frac{\overline{g}'^2}{4} I \leq -Q \tag{76}$$

According to [4], the above inequality has a feasible solution which can be obtained by using the interior-point method. Using (76) in (75) yields

$$\dot{V}_n \leq -e^T Q e + K(g' N(\zeta) + 1)\dot{\zeta} + \sum_{j=1}^{n-1} Q_j^2 - \sum_{j=1}^{n-1} L_j^2 \left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) + \sum_{k=1}^{n} \frac{\sigma}{\gamma_k} \widetilde{W}_k^T \left(W_k - W_K^0\right) + \frac{\sigma}{\gamma_{s1}} \widetilde{W}_{s1}^T \left(W_{s1} - W_{s1}^0\right) + \frac{\sigma}{\gamma_H} \widetilde{W}_H^T \left(W_H - W_H^0\right) + \frac{\sigma}{\gamma_S} \widetilde{W}_S^T \left(W_S - W_S^0\right) - \sum_{k=1}^{n} C_k z_k^2 + \sum_{k=1}^{n} r_k \frac{\widetilde{W}_k^T \widetilde{W}_k}{4} + r_H \frac{\widetilde{W}_H^T \widetilde{W}_H}{4} + \frac{\|\varepsilon_S^*\|^2}{4} + R_2 \tag{77}$$

Theorem. Applying control law (71), virtual control signals (39), (51), (60), adaptive laws (41), (42), (52), (61), (72), (73), (74), and state observer (16) to system (1) under Assumption 1-4 and in the presence of input saturation and delay, guarantees semi-globally uniformly ultimately boundedness of all closed-loop signals and the output tracking error converges to a small neighborhood of the origin.

Proof. Using Young's inequality, the following inequalities can be obtained:

$$\sum_{k=1}^{n}\frac{\sigma}{\gamma_k}\widetilde{W}_k^T\left(W_k - W_K^0\right) \leq -\sum_{k=1}^{n}\frac{\sigma}{2\gamma_k}\widetilde{W}_k^T\widetilde{W}_k + \sum_{k=1}^{n}\frac{\sigma}{2\gamma_k}\|W_k^* - W_k^0\|^2 \tag{78}$$

$$\frac{\sigma}{\gamma_{S1}}\widetilde{W}_{S1}^T\left(W_{S1} - W_{S1}^0\right) \leq -\frac{\sigma}{2\gamma_{S1}}\widetilde{W}_{S1}^T\widetilde{W}_{S1} + \frac{\sigma}{2\gamma_{S1}}\|W_{S1}^* - W_{S1}^0\|^2 \tag{79}$$

$$\frac{\sigma}{\gamma_H}\widetilde{W}_H^T\left(W_H - W_H^0\right) \leq -\frac{\sigma}{2\gamma_H}\widetilde{W}_H^T\widetilde{W}_H + \frac{\sigma}{2\gamma_H}\|W_H^* - W_H^0\|^2 \tag{80}$$

$$\frac{\sigma}{\gamma_S}\widetilde{W}_S^T\left(W_S - W_S^0\right) \leq -\frac{\sigma}{2\gamma_S}\widetilde{W}_S^T\widetilde{W}_S + \frac{\sigma}{2\gamma_S}\|W_S^* - W_S^0\|^2 \tag{81}$$

Substituting (25), (44) and (78)-(81) into (77) yields

$$\begin{aligned}\dot{V}_n \leq & -e^TQe + K(g'N(\zeta)+1)\dot{\zeta} + \sum_{j=1}^{n-1}L_j^2\left(\frac{1}{\pi_{j+1}} - \frac{1}{2}\right) - \sum_{k=1}^{n}C_k z_k^2 - \\ & \sum_{k=1}^{n}\left(\frac{\sigma}{2\gamma_k} - \frac{5}{4}r_k\right)\widetilde{W}_k^T\widetilde{W}_k - \frac{\sigma}{2\gamma_{S1}}\widetilde{W}_{S1}^T\widetilde{W}_{S1} - \left(\frac{\sigma}{2\gamma_H} - \left(u_m + \frac{1}{4}\right)r_H\right)\widetilde{W}_H^T\widetilde{W}_H - \\ & \frac{\sigma}{2\gamma_S}\widetilde{W}_S^T\widetilde{W}_S + \sum_{j=1}^{n-1}Q_j^2 + \sum_{k=1}^{n}\frac{\sigma}{2\gamma_k}\left\|W_k^* - W_k^0\right\|^2 + \frac{\sigma}{2\gamma_{S1}}\|W_{S1}^* - W_{S1}^0\|^2 + \\ & \frac{\sigma}{2\gamma_S}\|W_S^* - W_S^0\|^2 + \frac{\sigma}{2\gamma_H}\|W_H^* - W_H^0\|^2 + \frac{\left\|\varepsilon_S^*\right\|^2}{4} + \frac{\|\varepsilon_{S1}^*\|^2}{4} + \sum_{j=1}^{n}r_j\left(W_j^{*T}W_j^*\right) + \\ & \|\varepsilon^*\|^2 + u_m\|\varepsilon_H^*\|^2 + u_m r_H W_H^{*T}W_H^* + \frac{5}{4}\|D^*\|^2 + \frac{\left(\overline{g}'\right)^2}{4}\end{aligned} \tag{82}$$

Definition 2. Similar to the approach proposed in [6] and considering boundedness of y_r and $\dot{y}_r$, the compact sets $\Psi_i \in R^{\left(\left(\sum_{j=1}^{i}r_j\right)+3i+r_{S1}-1\right)}$ are introduced

$$\begin{aligned}\Psi_i := \Bigg\{ & \sum_{j=1}^{i}\left\|e_{j+1}\right\|^2 + \sum_{j=1}^{i}z_j^2 + \sum_{j=2}^{i}L_{j-1}^2 + \sum_{j=2}^{i}\frac{\widetilde{W}_j^T\widetilde{W}_j}{\gamma_j} + \frac{\widetilde{W}_{S1}^T\widetilde{W}_{S1}}{\gamma_{S1}} + \\ & \frac{\widetilde{W}_H^T\widetilde{W}_H}{\gamma_H} + \frac{\widetilde{W}_S^T\widetilde{W}_S}{\gamma_S} \leq 2\varpi_i\Bigg\}, 1 \leq i \leq n\end{aligned} \tag{83}$$

where ϖ_i denotes the positive constant for all initial conditions which satisfy the above inequality. Since Ψ_i is a compact set for continuous functions Q_i defined on

$R^{\left(\left(\sum_{j=1}^{i} r_j\right)+3i+r_{S1}-1\right)}, 1 \le i \le n-1$, then there exist positive constants $M_i, 1 \le i \le n-1$ such that inequalities $|Q_i| \le M_i, 1 \le i \le n-1$ are satisfied.

Using the above inequalities in (82), $\dot{V}_n$ can be written as

$$\dot{V}_n \le -\mu V_n + K(g' N(\zeta) + 1)\dot{\zeta} + \eta \tag{84}$$

where positive constants η and μ are defined as

$$\mu = min\left\{\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}, 2C_i, \left(\frac{2}{\pi_{k+1}} - 1\right), 2\gamma_H\left(\frac{\sigma}{2\gamma_H} - \left(u_m + \frac{1}{4}\right) r_H\right), 2\gamma_i\left(\frac{\sigma}{2\gamma_i} - \frac{5}{4}\right) r_i, \sigma\right\}, 1 \le i \le n, 1 \le k \le n-1 \tag{85}$$

$$\eta = \sum_{k=1}^{n} \frac{\sigma}{2\gamma_k}\|W_k^* - W_k^0\|^2 + \frac{\sigma}{2\gamma_{S1}}\|W_{S1}^* - W_{S1}^0\|^2 + \frac{\sigma}{2\gamma_S}\|W_S^* - W_S^0\|^2 + \sum_{j=1}^{n-1} M_j^2 + \frac{\sigma}{2\gamma_H}\|W_H^* - W_H^0\|^2 + \sum_{j=1}^{n} r_j\left(W_j^{*T} W_j^*\right) + \frac{\|\varepsilon_S^*\|^2}{4} + \frac{\|\varepsilon_{S1}^*\|^2}{4} + \|\varepsilon^*\|^2 + u_m\|\varepsilon_H^*\|^2 + u_m r_H W_H^{*T} W_H^* + \frac{5}{4}\|D^*\|^2 + \frac{\left(\overline{g'}\right)^2}{4} \tag{86}$$

$\lambda_{\min}(Q)$ and $\lambda_{\max}(P)$ denote the minimum eigenvalue of $Q \in R^{n\times n}$ and the maximum eigenvalue of $P \in R^{n\times n}$, respectively. Multiplying both sides of inequality (84) by $exp(-\mu t)$ and integrating over range $\left[0, t_f\right]$, yields

$$\frac{1}{2} z_1^2 \le V_n \le \frac{\eta}{\mu} + V_n(0) + \int_0^{t_f} e^{-\mu(t_f - t)} K(g' N(\zeta) + 1)\dot{\zeta} dt \tag{87}$$

Using (87) and Lemma 1, we conclude that all closed-loop signals including the output tracking error and the state observation error vector are bounded. Additionally, according to Lemma 1, we have $\left|\int_0^{t_f} e^{-\mu(t_f - t)} K(g' N(\zeta) + 1)\dot{\zeta} dt\right| \le \overline{C}$, where $\overline{C}$ is a positive constant. Using this inequality and (87), we have

$$|z_1| \le \sqrt{2\left(V_n(0) + \eta/\mu + \overline{C}\right)} \tag{88}$$

From (88), it is clear that $\sqrt{2\left(V_n(0) + \eta/\mu + \overline{C}\right)}$ can be decreased by increasing μ. Therefore the output tracking error, z_1, can be decreased by choosing the design parameter μ appropriately.

In what follows the steps required for designing the proposed controller and some guidelines for choosing the design parameters are provided.

1) Choose a positive vector $k = [k_1, \dots, k_n]^T$ to make matrix A Hurwitz.
2) Select a positive symmetric matrix $Q \in R^{n\times n}$ and solve (76) to obtain the positive symmetric matrix $P \in R^{n\times n}$.
3) Construct the appropriate Neural Networks with Gaussian functions given by (8) to estimate $\hat{F}_i\left(\underline{\hat{X}_i}\right), 1 \le i \le n$.
4) Select positive values for design parameters γ_i, γ_{S1}, γ_S, γ_H, σ, π_{j+1}, β, ϑ, K such that $\frac{\sigma}{\gamma_i} \ge \frac{5}{2} r_i, 1 \le i \le n$, $\pi_{j+1} < 2, 1 \le j \le n-1$, and $\frac{\sigma}{\gamma_H} \ge \left(\frac{1}{2} + 2u_m\right) r_H$.

Choosing large values for C_i and $k_i, 1 \le i \le n$ decreases the control surface errors, $|z_i|$, and estimation error e_i, but results in an aggressive control action. In order to reduce the growth rate of ζ and avoid oscillations in the Nussbaum function, it is recommended to choose high values for K. Set each elements of design vectors W_{S1}^0, W_S^0, $W_H^0, W_i^0, 1 \le i \le n$ to a positive constant. It is worth mentioning that these fixed vectors do not have significant effects on the closed-loop tracking performance.

5. Simulation results

In this section, two numerical examples are presented to verify the effectiveness of the proposed adaptive NN controller.

Example1: Consider the SISO nonlinear system described by the following differential equations with unknown control direction and subjected to external disturbances:

$$\begin{cases} \dot{x}_1 = g_1 x_2 - \frac{x_1}{1+x_1^4} + d_1(t) \\ \dot{x}_2 = \frac{7u(t-\tau)}{15+tanh(1+x_1)} - x_2 exp(-x_1^2) + d_2(t) \\ y = x_1 \end{cases} \quad (89)$$

where $\underline{x}_2 = [x_1, x_2]^T \in R^2$ is the states vector. $g_1 = 0.3$ and input time delay τ is set to 0.35. The unknown bounded disturbances $d_1(t)$ and $d_2(t)$ are selected to be $0.2sin\left(\frac{t}{3}\right)$ and $-0.1cos\left(\frac{t}{6}\right)$, respectively. It is assumed that the second state variable is unavailable and $u_m = 10$ is considered as the upper bound of the input. The reference signal is set to $y_r = cos(t)$ and $\underline{x}_2(0) = [1.5, 1]^T$ is considered as the initial condition of the states vector. For estimating the nonlinear functions, the NN approximator, $W_{s1}^T S_{s1}(Z_{s1})$, with input vector $Z_{s1} = [X_1]$ containing 4 nodes with centers $\mu_{s1,r_{s1}}(r_{s1} = 1, \dots, 4)$ and width $\sigma_{s1,r_{s1}} = 8, (r_{s1} = 1, \dots, 4)$ evenly spaced in $[-10,10]$ has been considered. To approximate the nonlinear functions $F_1(.)$, $F_2(.)$ and $H(.)$, three NNs, $W_i^T S_i(Z_k), i = 1,2, H$ with input vector $Z_k =$

$\left[X_1,\hat{X}_2\right]^T$containing 16 nodes with centers $\mu_{i,r_i}(r_i = 1,\dots,16)$ and width $\sigma_{i,r_i} = 8, (r_i = 1,\dots,16)$ evenly spaced in $[-10,10]\times[-10,10]$ are selected. Similarly, the NN $W_S^T S_S(Z_S)$ with input vector $Z_s = \left[X_1,\hat{X}_2,W_H,\int_{t-\tau}^{t} u(t)dt\right]^T$ containing 256 nodes with centers $\mu_{s,r_S}(r_s = 1,\dots,256)$ and width $\sigma_{s,r_s} = 8, (r_s = 1,\dots,256)$ evenly spaced in $[-10,10]\times[-10,10]\times[-10,10]\times[-10,10]$ is chosen.

The observer for this example has been designed based on (16). Furthermore, the controller has been applied by utilizing (70). The virtual control signals are calculated by utilizing (39), (51), (60) and to update the unknown parameters adaptive laws (41), (42), (52), (61), (72), (73),and (74) have been used.

The above mentioned equations for this example are provided below.

Equation used for the observer is as follows:

$$\begin{cases}\dot{\hat{X}}_1 = \hat{X}_2 + W_1^T S_1\left(\underline{\hat{X}_2}\right) + k_1\left(y-\hat{X}_1\right)\\ \dot{\hat{X}}_2 = W_H^T S_H\left(\underline{\hat{X}_2}\right)u(t-\tau) + W_2^T S_2\left(\underline{\hat{X}_2}\right) + k_2\left(y-\hat{X}_1\right)\end{cases} \tag{90}$$

Furthermore, the control action is given by

$$\nu = -\frac{\left(W_H^T S_H\left(\underline{\hat{X}_2}\right)\right)}{\left(W_H^T S_H\left(\underline{\hat{X}_2}\right)\right)^2+\vartheta}\left(C_2 z_2 + W_S^T S_S(\psi) + z_1 + 3z_2 + W_2^T S_2\left(\underline{\hat{X}_2}\right) + k_2 e_1 + \left(\dot{W}_H^T S_H\left(\underline{\hat{X}_2}\right)\right)\int_{t-\tau}^{t} u(s)ds - \dot{\omega}_2\right) - \beta\lambda sign\left(W_H^T S_H\left(\underline{\hat{X}_2}\right)\right) \tag{91}$$

where ω_2 and α_1 are obtained from the following equations:

$$\dot{\omega}_2\pi_2 + \omega_2 = \alpha_1\,, \omega_2(0) = \alpha_1(0) \tag{92}$$

$$\alpha_1 = N(\zeta)(C_1 z_1 + 5z_1 + W_{S1}^T S_{s1}(X_1) - \dot{y}_r) \tag{93}$$

and ζ satisfies the following differential equation:

$$\dot{\zeta} = \frac{z_1}{K}(C_1 z_1 + 5z_1 + W_{S1}^T S_{s1}(X_1) - \dot{y}_r) \tag{94}$$

The corresponding adaptation laws are given by

$$\dot{W}_{S1} = \gamma_{S1} z_1 S_{S1}(X_1) - \sigma(W_{S1} - W_{S1}^0) \tag{95}$$

$$\dot{W}_1 = \gamma_1 z_1 S_1(\underline{\hat{X}}_2) - \sigma(W_1 - W_1^0) \tag{96}$$

$$\dot{W}_2 = \gamma_2 z_2 S_2(\underline{\hat{X}}_2) - \sigma(W_2 - W_2^0) \tag{97}$$

$$\dot{W}_H = \gamma_H z_n S_H(\underline{\hat{X}}_2) - \sigma(W_H - W_H^0) \tag{98}$$

$$\dot{W}_S = \gamma_S z_n S_S(\psi) - \sigma(W_S - W_S^0) \tag{99}$$

The design parameters are set to $C_1 = 2, C_2 = 4$, $\pi_2 = 0.5, \sigma = 6.5$, $\gamma_s = 0.1$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\gamma_H = 0.015$, $\gamma_{s1} = 0.1$, $\beta = 7$, $\vartheta = 0.3$ and each element of design vectors $W_1^0, W_2^0, W_{s1}^0, W_H^0$ and W_S^0 is fixed to 0.1. Observer design parameters are chosen to be $k_1 = 30, k_2 = 50$.

The tracking performance of the proposed control scheme has been shown in Fig. 1. As can be seen, the reference signal has been tracked quite well. The corresponding control action has been shown in Fig. 2. The system states and their estimates are shown in Figs. 3 and 4. Variations of the Nussbaum gain function and its argument are shown in Fig. 5.

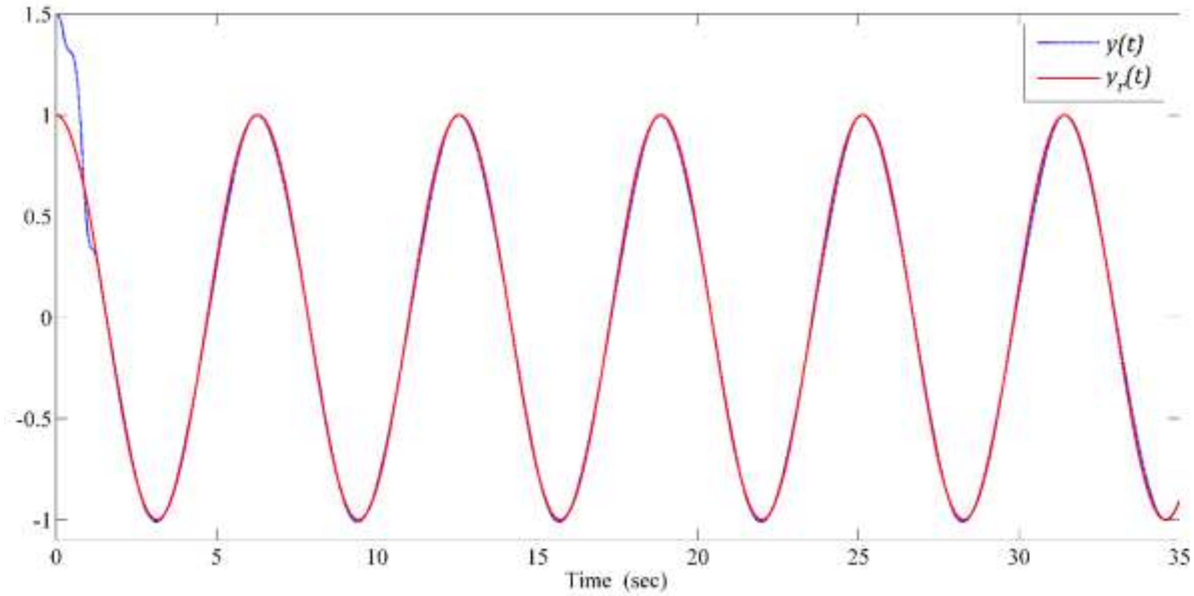

Fig. 1. The system output and the reference signal.

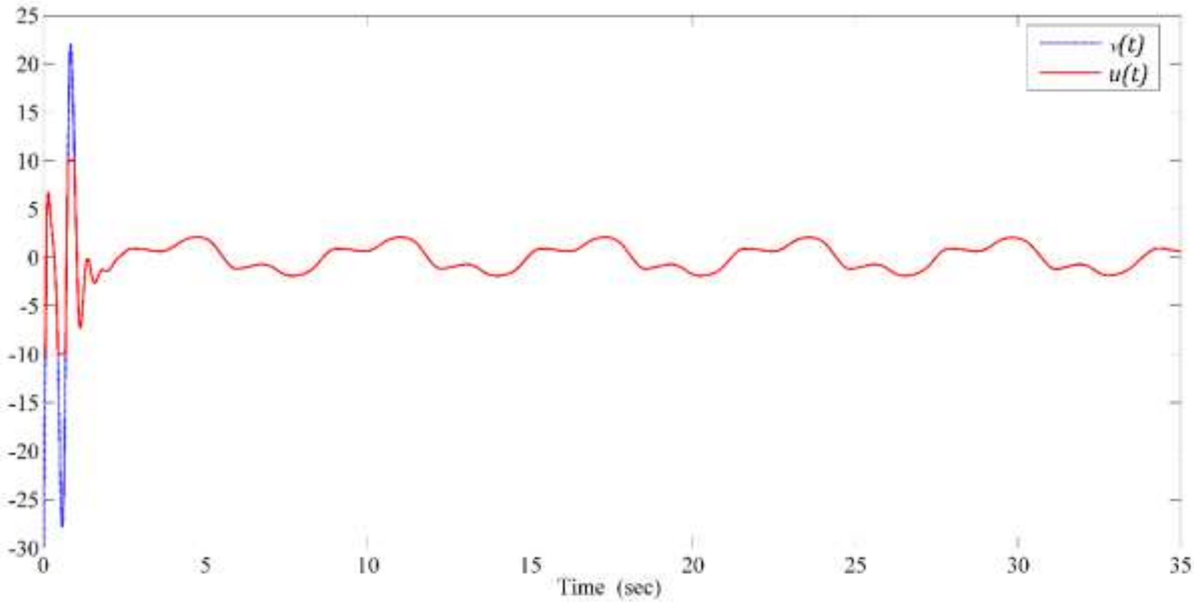

Fig. 2. The control signal and the system input.

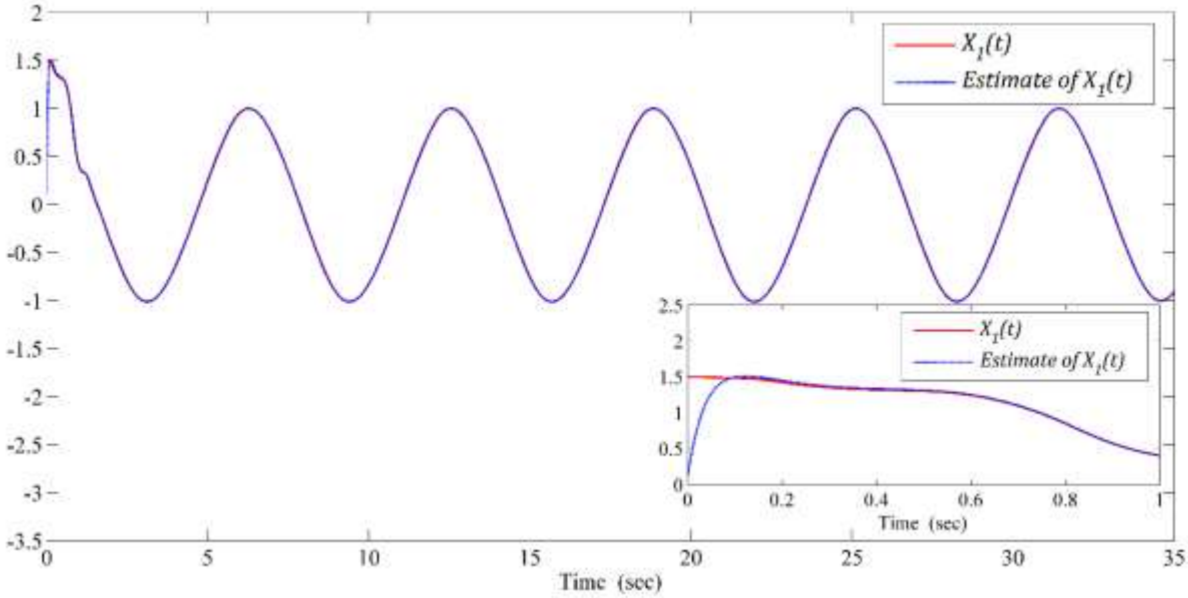

Fig. 3. $X_1(t)$ and its estimate.

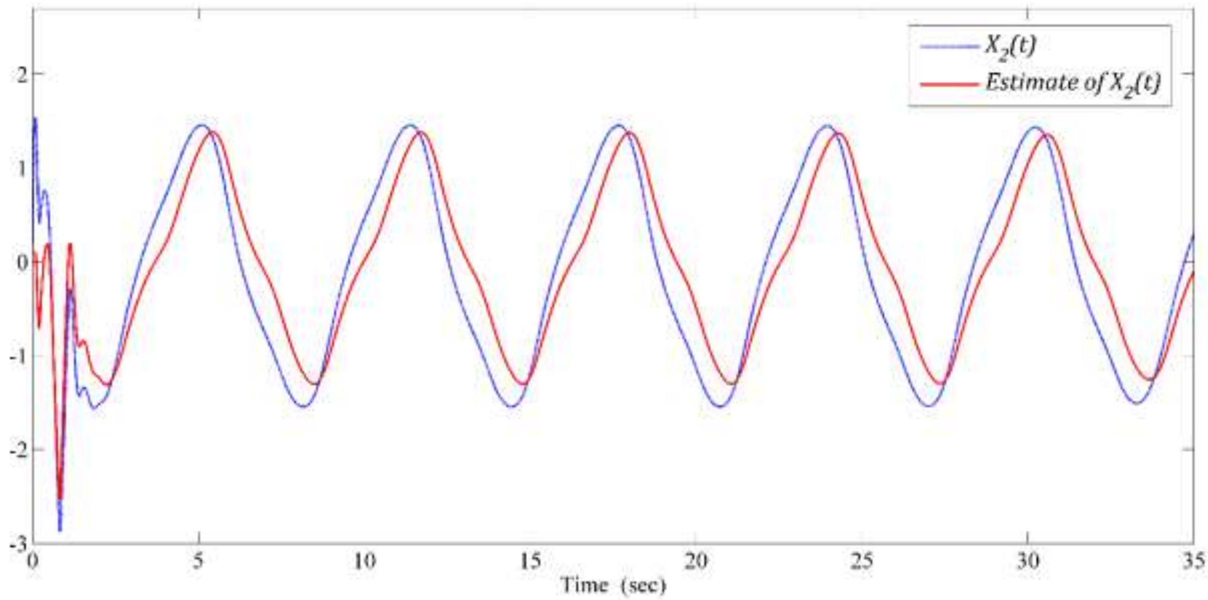

Fig. 4. $X_2(t)$ and its estimate.

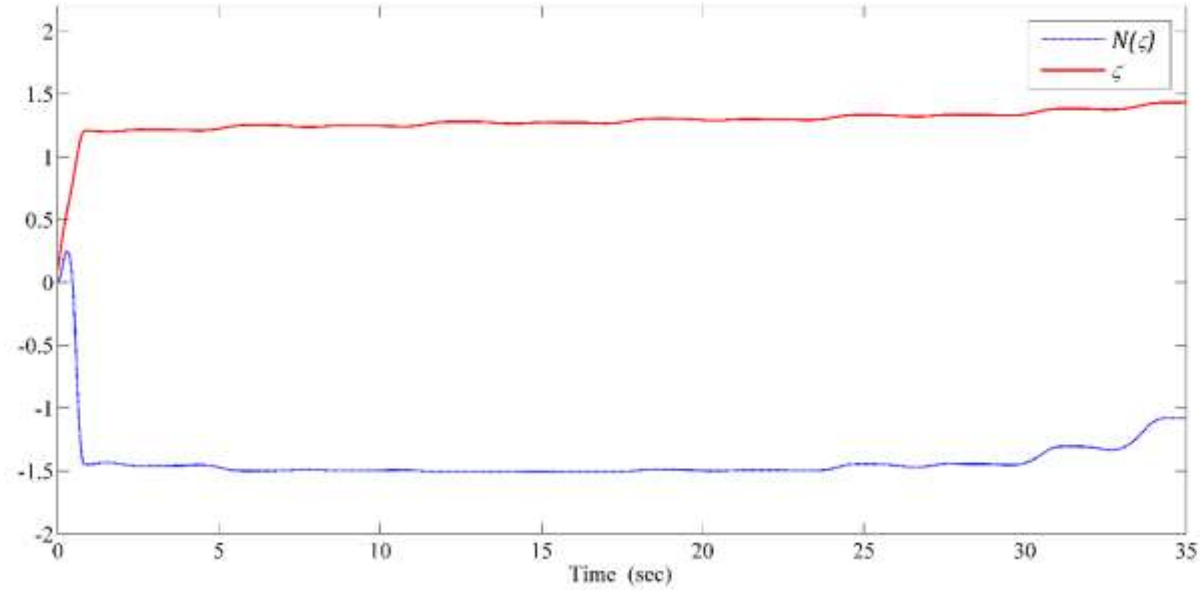

Fig. 5. The Nussbaum gain function and its argument.

Example 2: Consider the following two cascaded stirred isothermal reactors depicted in Fig. 6. It is assumed that the m order chemical reaction, $mA \rightarrow B$ takes place in these reactors. The dynamical model of these continuous stirred tank reactors (CSTR) can be obtained via mass balance of species A as given below

$$\begin{cases} \frac{dC_{A2}(t)}{dt} = \frac{F_1}{V_2} C_{A1}(t) - \frac{F_2}{V_2} C_{A2}(t) - k_{r2} C_{A2}^m(t) + \frac{1}{V_2} d_1(t) \\ \frac{dC_{A1}(t)}{dt} = \frac{F_i}{V_1} C_{Ai}(t-\tau) - \frac{F_1}{V_1} C_{A1}(t) - k_{r1} C_{A1}^m(t) + \frac{1}{V_1} d_2(t) \\ y = C_{A2}(t) \end{cases} \tag{100}$$

where $C_{A1}(t)$ and $C_{A2}(t)$ are the concentrations of species A in the first and second reactor, respectively. Constants F_i, F_1 and F_2 are the flow rates of liquid streams as shown in Fig. 6. V_1 and V_2 are volumes of the two reactors and positive constants k_{r1} and k_{r2} are the isothermal reaction coefficients of the first and second reactors, respectively. Variables $d_1(t)$ and $d_2(t)$ are unmeasured inlet mole flow to the first and second reactors and considered as external disturbances. $C_{Ai}(.)$ is the inlet concentration of species A into the first reactor and considered as the manipulated variable. It is worth mentioning that variable $C_{Ai}(.)$ can be calculated easily through a mass balance on the mixing zone of the two adjustable diluted and concentrated streams which leads to

$$\begin{cases} C_{Ai}(t) = \frac{F_C(t)C_{AC}+F_D(t)C_{AD}}{F_C(t)+F_D(t)} \\ F_C(t) + F_D(t) = F_i \end{cases} \quad (101)$$

It is assumed that only $C_{A2}(t)$ is measured and there is a known constant transportation delay $\tau = \frac{V_{pipe}}{F_i}$ for $C_{Ai}(.)$ entering to the first reactor. Values of the constants appearing in (100) and (101), are given in Table 1.

Table 1

Values of parameters for the two series of continuous stirred tank reactors

Parameter	**Symbol**	**Value**
The first reactor volume	V_1	$25\ lit$
The second reactor volume	V_2	$25\ lit$
The first reactor inlet flow stream	F_i	$10\ (lit/min)$
The second reactor inlet flow stream	F_1	$10\ (lit/min)$
The second reactor outlet flow stream	F_2	$10\ (lit/min)$
The isothermal reaction coefficient in the first reactor	k_{r1}	$0.428\ (\frac{Lit}{mol.min})$
The isothermal reaction coefficient in the second reactor	k_{r2}	$0.454(\frac{Lit}{mol.min})$
Order of the reaction	m	2
Concentration of the diluted stream	C_{AD}	$0\ (\frac{mol}{Lit})$
Concentration of the concentrated stream	C_{AC}	$20\ (\frac{mol}{Lit})$
The steady state concentration in the first reactor	C_{A1SS}	$2.56\ (\frac{mol}{Lit})$
The steady state concentration in the second reactor	C_{A2SS}	$1.15\ (\frac{mol}{Lit})$
The transmission delay	τ	0.18 min

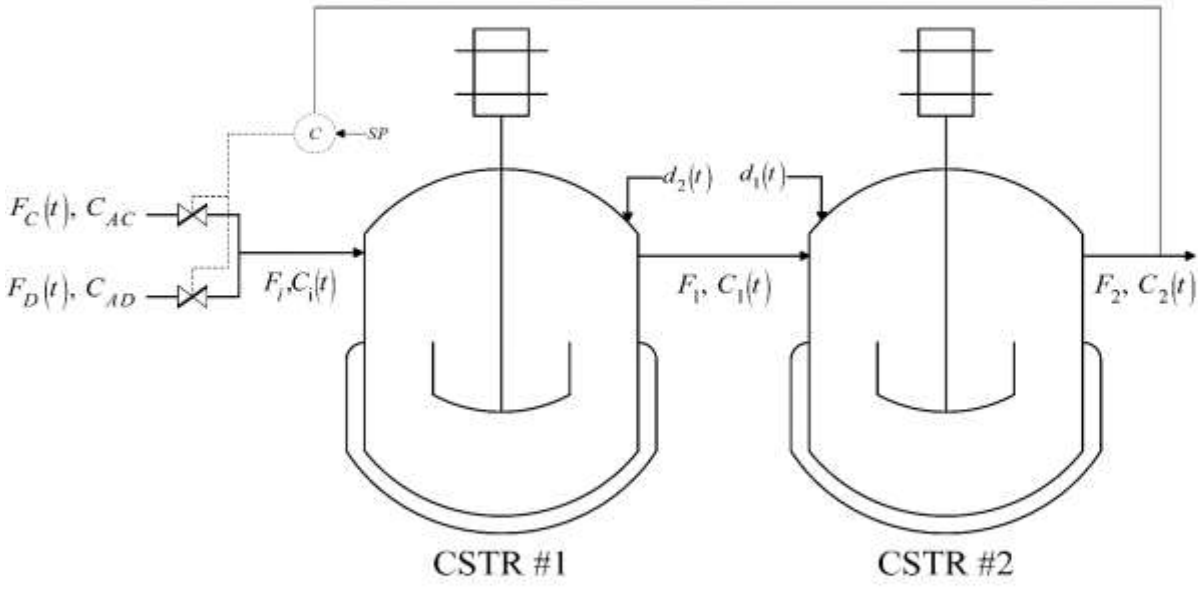

Fig. 6. Schematic diagram of two continuous stirred tank reactors with input delay.

In order to examine the capability of the proposed controller in load rejection, $d_2(t)$ is set to 6 ($\frac{mol}{min}$) during the simulation and the first disturbance $d_1(t)$ is subjected to step wise changes as given by (102)

$$d_1(t) = \begin{cases} 0 & t < 20\ min \\ 5 & t \geq 20\ min \end{cases} \tag{102}$$

It is desired that the concentration of the second reactor tracks the reference trajectory $C_{A2ref} = 1.15$ ($\frac{mol}{Lit}$). The initial conditions of both reactors concentrations, their estimates, and other required parameters are: $\begin{bmatrix} C_1(0) \\ C_2(0) \end{bmatrix} = \begin{bmatrix} 5.05 \\ 3.10 \end{bmatrix}$, $\begin{bmatrix} \hat{C}_1(0) \\ \hat{C}_2(0) \end{bmatrix} = \begin{bmatrix} 2.65 \\ 2.10 \end{bmatrix}$, $W_{s1}(0) = 0.1$, $W_1(0) = W_2(0) = 0.13$, $W_S(0) = 0$, $W_H(0) = 0.5$, $\zeta(0) = 0$, $\lambda(0) = 0$.

To estimate the nonlinear functions, the NN approximator, $W_{s1}^T S_{s1}(Z_{s1})$, with input vector $Z_{s1} = [C_{A2}]$ containing 4 nodes with centers $\mu_{s1,r_{s1}}(r_{s1} = 1, \ldots, 4)$ and width $\sigma_{s1,r_{s1}} = 6, (r_{s1} = 1, \ldots, 4)$ evenly spaced in $[-8,8]$ has been considered. To approximate the nonlinear functions $F_i(.)$, $i = 1,2$ and H, three NNs, $W_i^T S_i(Z_k), i = 1,2$ and H, with input vector $Z_k = \left[C_{A2}, \hat{C}_{A1}\right]^T$containing 16 nodes with centers $\mu_{i,r_i}(r_i = 1, \ldots, 16)$ and width $\sigma_{i,r_i} = 6, (r_i = 1, \ldots, 16)$ evenly spaced in $[-8,8] \times [-8,8]$ are selected. Similarly, the NN $W_S^T S_S(Z_S)$ with input vector $Z_s = \left[C_{A2}, \hat{C}_{\mathrm{A}1}, W_H, \int_{t-\tau}^{t} u(t)dt\right]^T$ containing 256 nodes with centers $\mu_{s,r_s}(r_s = 1, \ldots, 256)$ and width $\sigma_{s,r_s} = 6, (r_s = 1, \ldots, 256)$ evenly spaced in $[-8,8]$ $[-8,8] \times [-8,8] \times [-8,8]$ is chosen.

The rest of design parameters are given as follows: $k_1 = 12$, $k_2 = 19.5$, $K = 45$, $C_1 = 7$, $C_2 = 5$, $\pi_2 = 0.65$, $\sigma = 6.8$, $\gamma_{s1} = 0.15$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $\gamma_S = 0.1$, $\gamma_H = 0.015$, $\beta = 1.5$, $\vartheta = 1$ and $W_j^0 = 0.08, j = 1, 2, S_1, S, H$.

The process output and the desired trajectory are demonstrated in Fig. 7. Fig. 8 and Fig. 9, represent variations of species A concentrations and their estimations in the first and second reactors. Variations of the manipulated variable C_{Ai} and the applied control signal are shown in Fig. 10. Fig. 11 depicts the Nussbaum function and its argument.

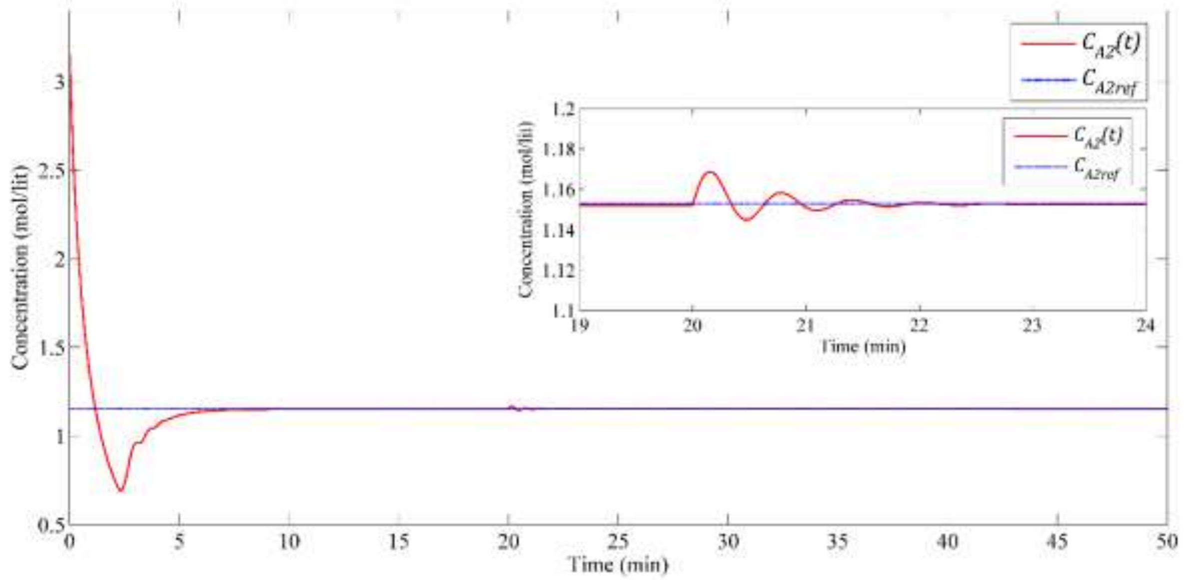

Fig. 7. Concentration of the second reactor (red) and its desired trajectory (blue).

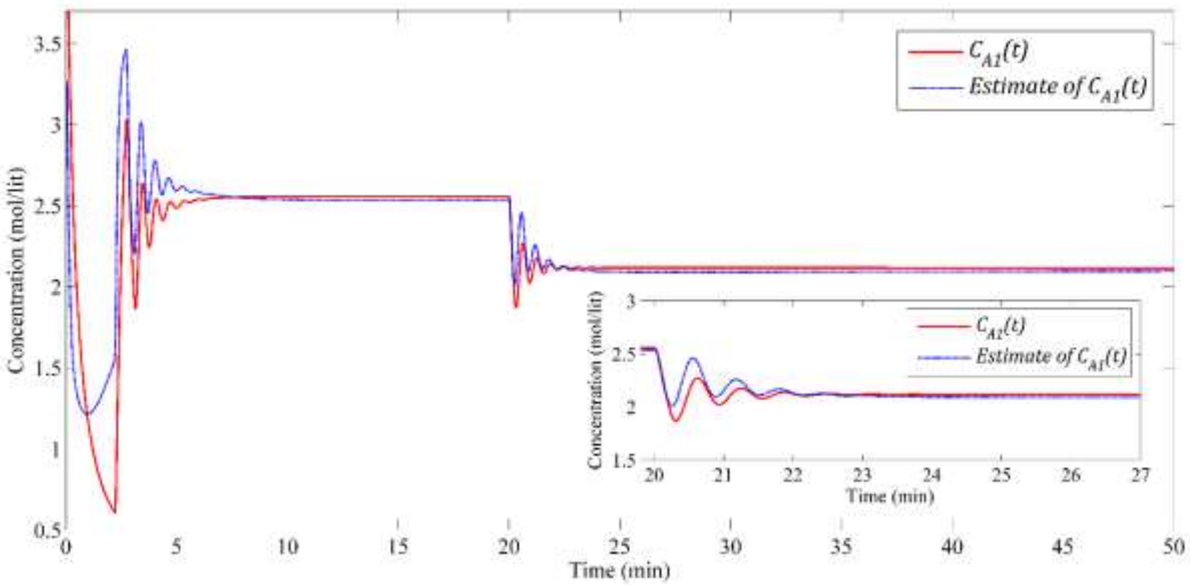

Fig. 8. Variations of the first reactor concentration (red) and its estimate (blue).

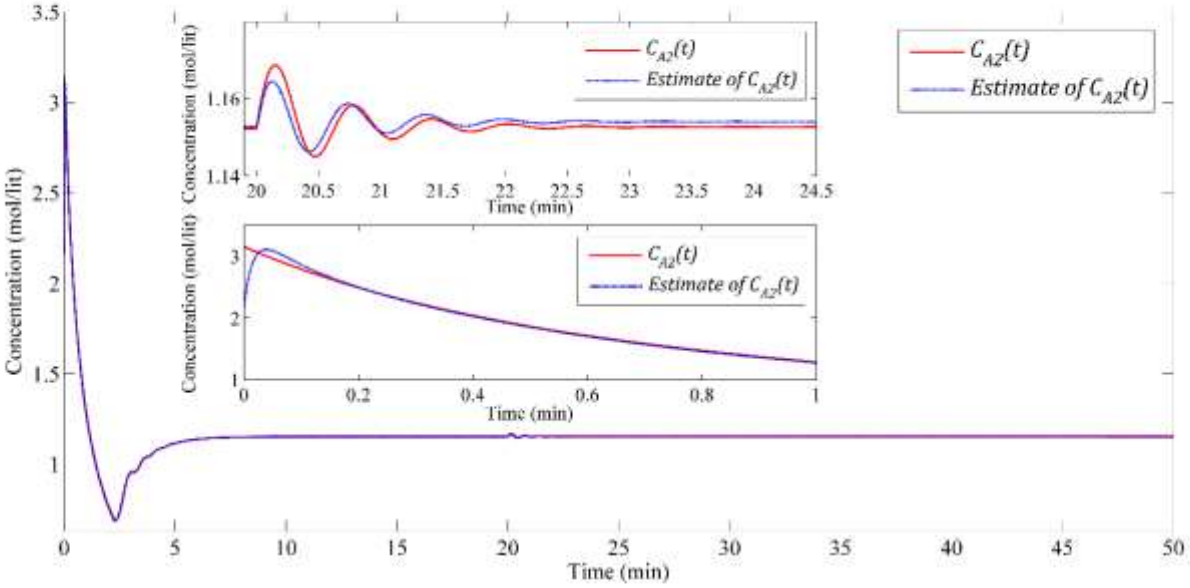

Fig. 9. Variations of the second reactor concentration (red) and its estimation (blue).

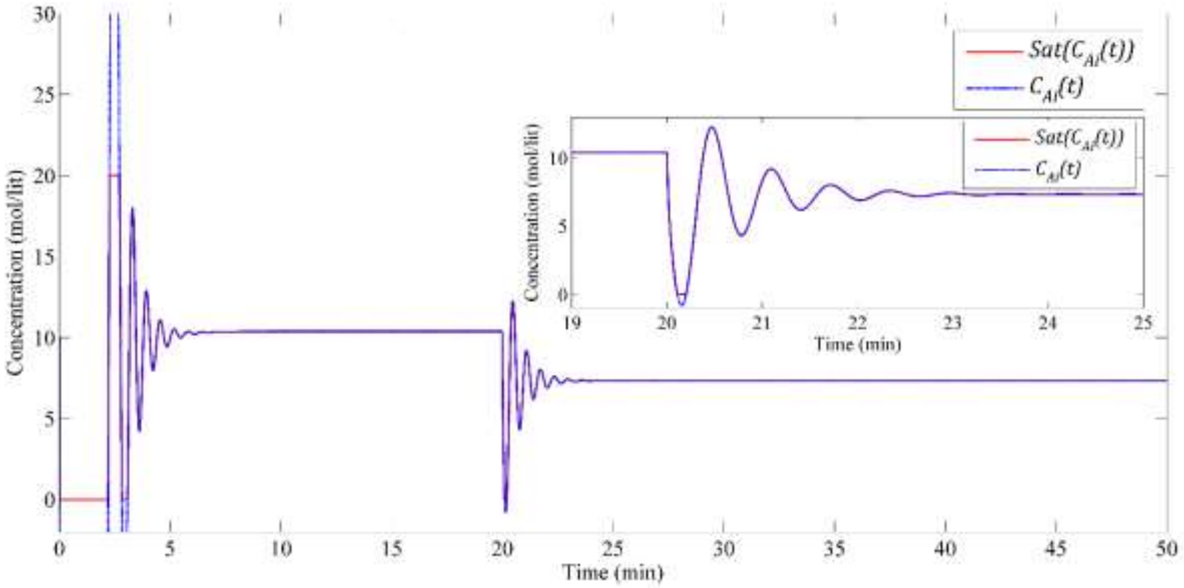

Fig. 10. Variations of process input variable C_{Ai} (blue) and the control signal (red).

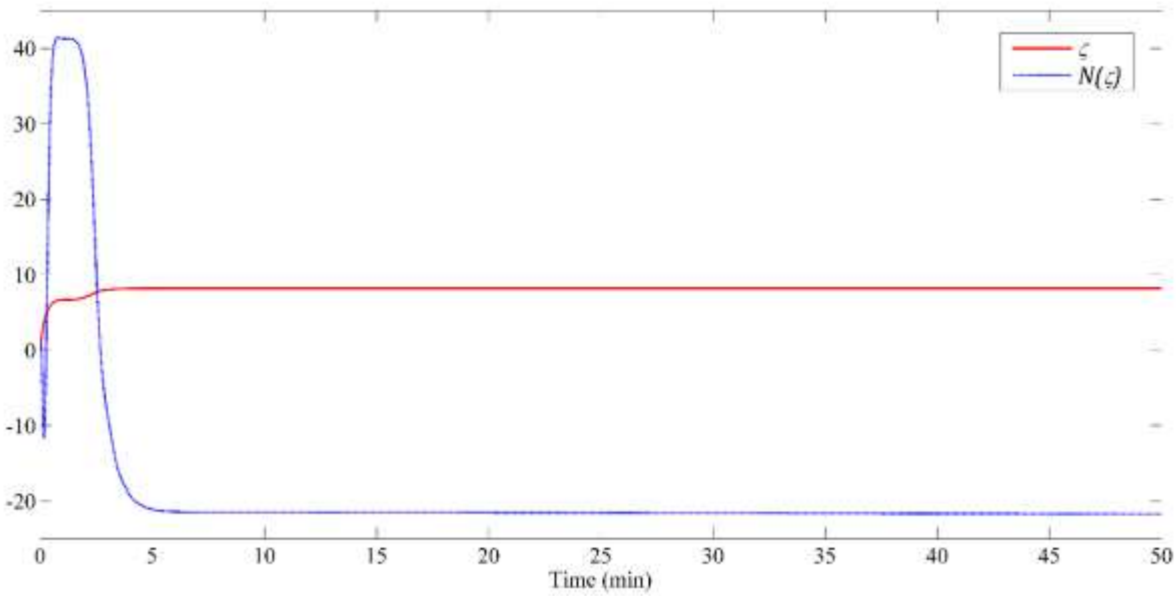

Fig. 11. Variations of the Nussbaum function (blue) and its argument (red).

6. Conclusion

As stated in the introduction section there is no singularity free controller scheme proposed in the literature that can control an uncertain nonlinear system with unmeasured states subject to input saturation, input delay, and unknown control directions. In this paper, an observer-based adaptive singularity free output feedback NN controller is proposed for a class of nonlinear strict-feedback systems with unknown control directions in the presence of system input time delay and saturation. The proposed design method does not require a priori knowledge of the unknown virtual control coefficients signs. The controller singularity problem is avoided by employing a novel approach. The effect of input delay has been compensated utilizing an integral term in the last step of the controller design. "The explosion of complexity" occurring in the traditional backstepping technique has been avoided by utilizing the DSC technique in the controller design. It has been shown that all closed-loop signals remain semi-globally uniformly ultimately bounded (SGUUB) and the output tracking error converges to a small neighborhood of the origin by choosing the design parameters appropriately. Some guidelines for designing the proposed controller and selecting the design parameters are provided. Effectiveness of the proposed control scheme has been demonstrated via simulation study. In one of the simulation examples a chemical reactor system has been considered to show the application of the proposed control scheme. Simulation results show that all desired objectives have been achieved satisfactorily.

7. Future work

To make the proposed control scheme more suitable for the practical applications, actuator failure should be also taken into account. Extending the proposed algorithm to multi-input multi-output interconnected large-scale nonlinear systems is another aspect which improves the capability of the proposed control scheme. These two subjects will be considered as future works.

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